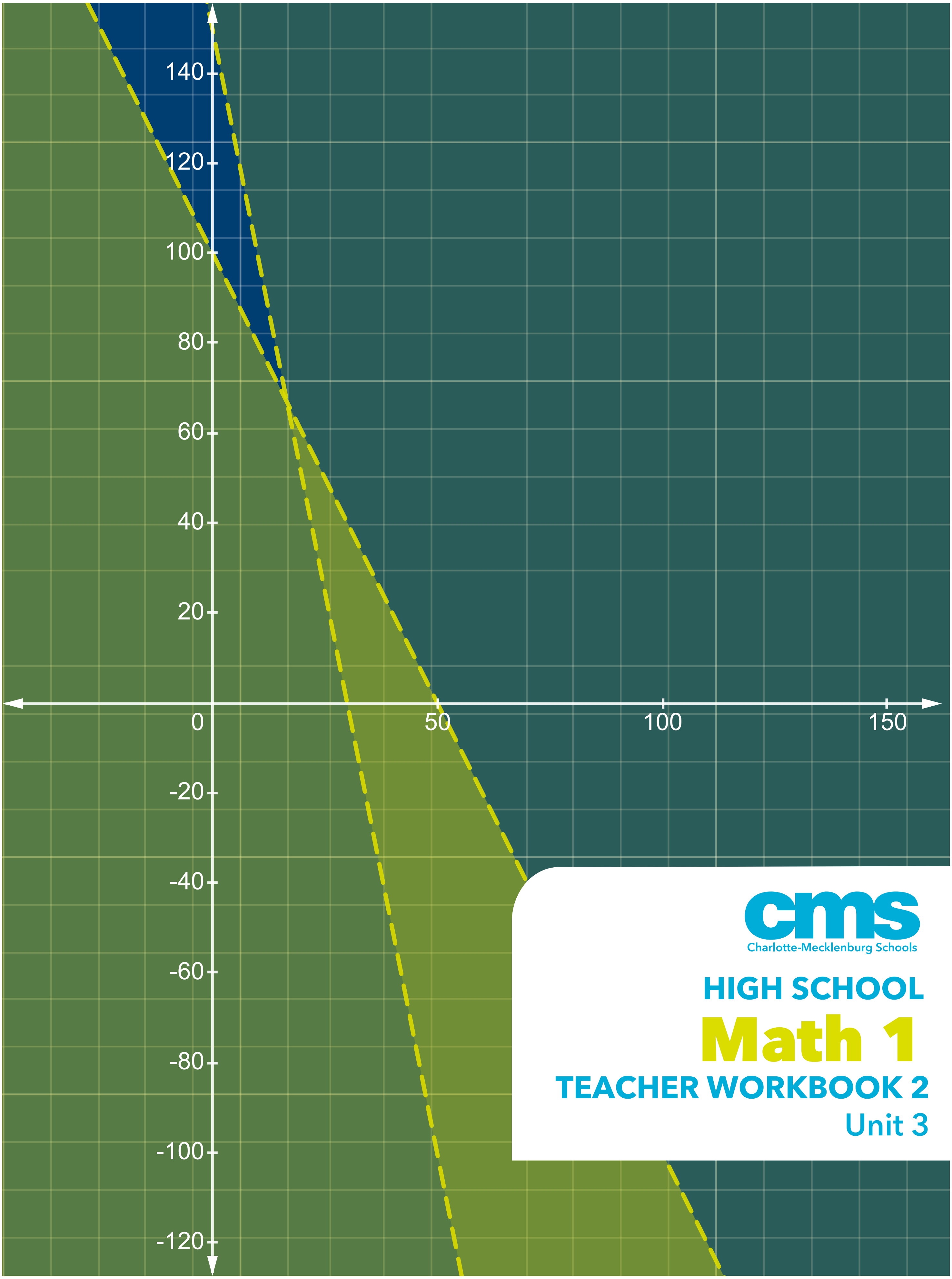


Charlotte-Mecklenburg Schools

HIGH SCHOOL

Math 1

TEACHER WORKBOOK 2

Unit 3

Table of Contents

Math 1

This set of instructional resources aims to provide a math curriculum that students and stakeholders can leverage to promote racial tolerance and oppose racism. Woven throughout the course are experiences for students and stakeholders to examine ideas of social justice, engage in current events, and expand and apply mathematics into everyday life. Reflection, student voice and agency, and high expectations are critical components of this curriculum. As a result, students consistently have opportunities to dig deeper into their worldviews and their identities as mathematicians. It is important to note that some of the topics may encourage passionate conversations and debate among students. Teachers should discuss these opportunities for student discourse during their planning meetings and recognize these possibilities when implementing the curriculum in their own classroom, leveraging their strong classroom cultures and inclusive classroom environments.

Unit 3: Coordinate Geometry and Systems of Linear Equations and Inequalities

In the previous unit, students wrote equations and inequalities to express constraints. They focused on algebraic, verbal, and tabular representations of these constraints. In this unit, students add graphical representations as a tool to continue exploring linear relationships. Some of these visual topics will be familiar: students studied slope-intercept form in grade 8 and learned to solve systems of equations by graphing. Other topics, like two-variable inequalities, will be entirely new.

In lessons 1–4, students are reminded that the graph of a line represents all solutions to an equation. They learn to write and to graph equations for lines in different forms (standard, slope-intercept, and point-slope), and see that the different forms make different features of the graphs most apparent. For instance, standard form ($ax + by = c$) allows students to quickly find the intercepts by setting one variable equal to zero and solving for the other. Slope-intercept form ($y = mx + b$) allows students to find the slope and *y*-intercept directly from the equation.

Lessons 5–6 introduce students to the fact that parallel lines have the same slope, while perpendicular lines have opposite reciprocal slopes. In both lessons, students practice writing equations for a line that is parallel/perpendicular to a given line and passes through a given point. Since the given information amounts to a slope and a point, using point-slope form is particularly useful. Students apply that understanding of parallel and perpendicular lines in Lesson 7 as they determine whether quadrilaterals in the coordinate plane are really squares, rectangles, parallelograms, and rhombuses. Students also find the length of line segments using first the Pythagorean theorem and eventually the distance formula and then use those lengths to calculate perimeter and area.

Lessons 8–12 are devoted to systems of equations. Students begin by building on the work they did in middle school, writing systems of equations to describe constraints and solving those systems by graphing. They are then introduced to the substitution method. In addition to the mechanics of substitution, students learn to think strategically about the expression they choose to isolate; for some systems, it could be more efficient to isolate the expression $2x$ rather than x. Students then learn to solve systems of equations by elimination, to explain why the steps taken to eliminate a variable are valid and productive, and to articulate how the process essentially entails writing a series of equivalent systems. Additionally, students reinforce their awareness that a system of equations could have one solution, no solutions, or infinitely many solutions.

Lessons 13–14 are Checkpoint Lessons. Stations include an opportunity for small-group instruction, practice and extension of this unit's material, and micro-modeling. One required station includes a mini-lesson on using coordinates to find the midpoint or endpoint of a line segment.

In Lessons 15–20, students rely on their understanding of equations and their graphs to explore two-variable inequalities. Students learn that a solution to a two-variable inequality is a pair of values that makes the inequality true, and a solution to a system of inequalities in two variables is any pair of values that make both inequalities in the system true. The solution set of a system of inequalities, they learn, can be best represented by graphing.

Lesson 21 occurs after administering the Unit 3 assessment and includes post-assessment activities. Taking this time to pause after the assessment to collect student reflection data through a survey and teacher conferences is a critical aspect of the course and building the classroom culture. The Student Survey is an opportunity to gather low-stakes, non-evaluative feedback for teachers to support equity and instructional pedagogy. Students will explore connections between mathematics and a subject or activity they engage in with their friends, family, and/or community.

Instructional Routines

Aspects of Mathematical Modeling: Lessons 13 & 14, 15, 17, 20

Card Sort: Lessons 12, 17

Collect and Display (MLR2): Lessons 2, 3, 6, 20

Compare and Connect (MLR7): Lessons 4, 8, 9, 17, 20

Critique, Correct, Clarify (MLR3): Lessons 10, 11

Discussion Supports (MLR8): Lessons 1, 3, 4, 6, 7, 8, 9, 12, 16, 18, 19

Graph It: Lessons 1, 3, 11, 12, 17, 18

Math Talk: Lessons 8, 9, 16

Notice and Wonder: Lessons 5, 10, 20

Poll the Class: Lesson 16

Round Robin: Lessons 1, 2, 7

Stronger and Clearer Each Time (MLR1): Lessons 5, 10, 16, 18

Take Turns: Lessons 3, 17, 19

Three Reads (MLR6): Lessons 1, 11, 12, 15, 19

Which One Doesn't Belong?: Lessons 1, 6, 12, 19

Lesson 1: Equations and Their Graphs

PREPARATION

Lesson Goals	Learning Targets
• Comprehend that the graph of a linear equation in two variables represents all pairs of values that are solutions to the equation. • Interpret points on a graph of a linear equation to answer questions about the quantities in context. • Use graphing technology to graph linear equations and identify solutions to the equations.	• When given the graph of a linear equation, I can explain the meaning of the points on the graph in terms of the situation it represents. • I understand how the coordinates of the points on the graph of a linear equation are related to the equation. • I can use graphing technology to graph linear equations and identify solutions to the equations.

Lesson Narrative

In Unit 2, students primarily used descriptions, expressions, and equations to represent relationships and constraints. In this lesson, they revisit the idea that graphs can be a useful way to represent relationships. Students are reminded that each point on a graph is a solution to an equation the graph represents. They analyze points on and off a graph and interpret them in context. In explaining correspondences between equations, verbal descriptions, and graphs, students hone their skill at making sense of problems (MP1).

In this lesson, students use Desmos to graph equations. Students were introduced to doing this in Unit 2, Lesson 18. This introduction could happen independently of that lesson as long as it precedes Activity 2 in this lesson.

What math language will you want to support your students with in this lesson? How will you do that?

Focus and Coherence

Building On	Addressing
NC.8.F.4: Analyze functions that model linear relationships. • Understand that a linear relationship can be generalized by $y=mx+b$. • Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two (x, y) values or a graph. • Construct a graph of a linear relationship given an equation in slope-intercept form. • Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and y-intercept of its graph or a table of values. **NC.8.F.5:** Qualitatively analyze the functional relationship between two quantities. • Analyze a graph determining where the function is increasing or decreasing; linear or non-linear. • Sketch a graph that exhibits the qualitative features of a real-world function. **NC.7.EE.4:** Use variables to represent quantities to solve real-world or mathematical problems. a. Construct equations to solve problems by reasoning about the quantities. • Fluently solve multistep equations with the variable on one side, including those generated by word problems. • Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. • Interpret the solution in context	**NC.M1.A-CED.2:** Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities. **NC.M1.A-REI.10:** Understand that the graph of a two variable equation represents the set of all solutions to the equation.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*10 minutes*)
- **Activity 2** (*15 minutes*)
  - Graphing technology is required: Acquire devices that can access Desmos (recommended) or other graphing technology. It is ideal if each student has their own device.
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U3.L1 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.7.EE.4.a

The purpose of this bridge is to elicit strategies students have for solving equations in one variable, especially equations involving an expression in parentheses being multiplied by a number. Developing fluency with these types of equations will be helpful when students substitute expressions in place of single variables and solve equations with expressions in parentheses. In particular, students might articulate that a productive first step in equation “a” is to divide each side by 10.

Student Task Statement

Solve each equation mentally.

a. $100 = 10(x-5)$

b. $300 = 30(x-5)$

c. $15 - 971 = x - 4 - 971$

d. $\frac{10}{7} = \frac{1}{7}(x-19)$

DO THE MATH	PLANNING NOTES

Warm-up: Hours and Dollars (*5 minutes*)

Instructional Routines: Which One Doesn't Belong?; Round Robin
Building On: NC.8.F.4; NC.8.F.5

This warm-up prompts students to carefully analyze and compare features of graphs of linear equations using the *Which One Doesn't Belong?* routine. In making comparisons, students have a reason to use language precisely (MP6). The activity also enables the teacher to hear the terminology students know and learn how they talk about characteristics of graphs.

WHICH ONE DOESN'T BELONG?	***What Is This Routine?*** Students are presented with four figures, diagrams, graphs, or expressions with the prompt: "Which one doesn't belong?" Typically, each of the four options "doesn't belong" for a different reason, and the similarities and differences are mathematically significant. Students are prompted to explain their rationale for deciding that one option doesn't belong and given opportunities to make their rationale more precise. ***Why This Routine?*** *Which One Doesn't Belong?* fosters a need to define terms carefully and use words precisely (MP6) in order to compare and contrast a group of mathematical objects or representations. Because there are no wrong answers, the focus is on student reasoning, and especially on students communicating their reasoning. This routine cultivates an inclusive classroom culture by prompting students to be creative thinkers, clear communicators, and good listeners.

The work here prepares students to reason about solutions to equations by graphing, which is the focus of this lesson.

Step 1

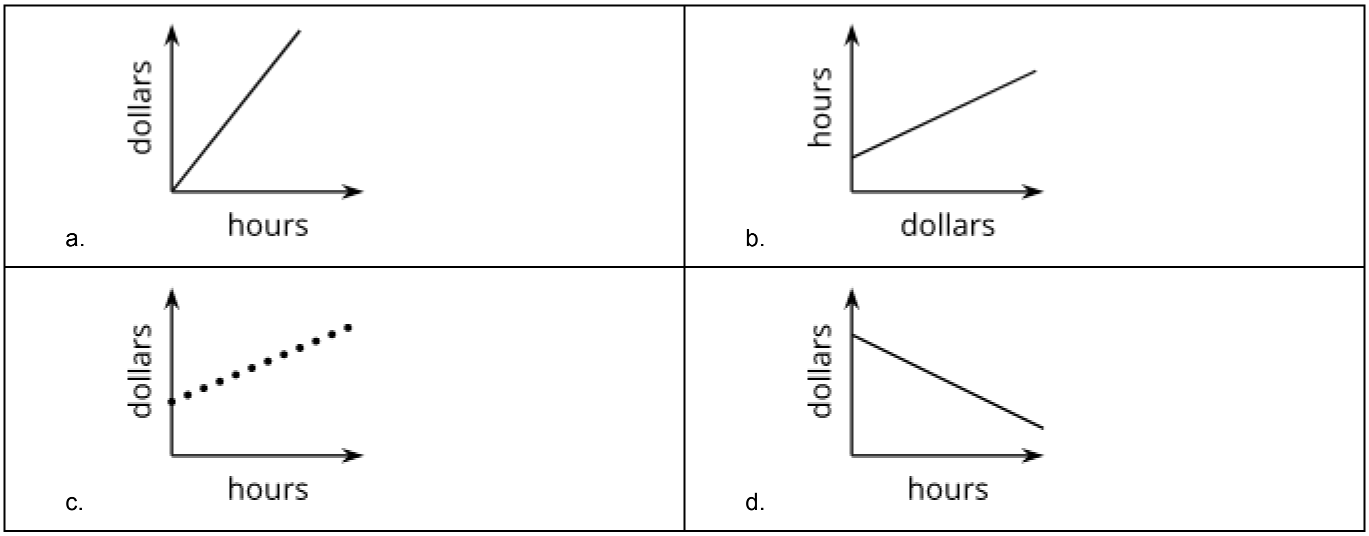
- Ask students to arrange themselves in small groups or use visibly random grouping.
- Display the graphs for all to see.
- Give students 1 minute of quiet think time and then time to share their thinking with their small group using the *Round Robin* routine. Ask each student to share with their small group their reasoning as to why a particular graph does not belong, and then ask the group to work together to find at least one reason each item doesn't belong.

ROUND ROBIN	***What Is This Routine?*** In small groups, students take turns sharing their rough draft response to an open-ended question while other group members refrain from comments or questions. A prop can be passed within each group to indicate whose turn it is to talk. After each student has had a turn to share, the group can ask questions of each other; then the teacher selects students to share with the whole class what their group members said.

	Why This Routine? Engaging a group of students in collaborative problem solving, with equitable inclusion of ideas, can be challenging due to normative social status issues that place higher value on some students' contributions over others. *Round Robin* allows all students to include their rough draft ideas for solving an open-ended problem without a subset of students dominating the conversation. Knowing all ideas will be shared should motivate all students to try at least one strategy to solve a problem on their own, critical for making sense of problems and persevering in solving them (MP1). The active sharing and listening involved in this routine also provides opportunity for constructing and critiquing viable arguments (MP3).

Student Task Statement

Which one doesn't belong? Explain your reasoning.



Step 2

- Ask each group to share one reason why a particular item does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given are correct.
- During the discussion, ask students to explain the meaning of any terminology they use, such as "y-intercept" or "negative slope." Also, press students on unsubstantiated claims. (For example, if a student claims that graph A is the only one with a slope greater than 1, ask them to explain or show how they know.)

DO THE MATH	PLANNING NOTES

Activity 1: A Family Reunion (*10 minutes*)

Instructional Routine: Three Reads (MLR6)	
Addressing: NC.M1.A-CED.2; NC.M1.A-REI.10	

Previously, students saw that an equation in two variables can have many solutions because there are many pairs of values that satisfy the equation. This activity illustrates that idea graphically. Students see that the coordinates of all points on the graphs are pairs of values that make the equation true, which means that they are all solutions to the equation.

They also see that, because the given equation models the quantities and constraints in a situation, not all points on the graph are meaningful. For example, only positive x or y values on the graph (that is, only points in the first quadrant of the coordinate plane) have meanings in this context, because chicken tenders and potato wedges cannot have negative values for their weight.

Note: this activity uses chicken tenders and potato wedges as the items purchased. If appropriate, substitute other items that may be more meaningful to students.

RESPONSIVE STRATEGY

Provide appropriate reading accommodations and supports to ensure student access to written directions, word problems, and other text-based content.

Supports accessibility for: Language; Conceptual processing

Step 1

- Ask students to arrange themselves in pairs, or use visibly random grouping, and provide access to calculators.
- Use the *Three Reads* routine to support comprehension of this word problem.

THREE READS	***What Is This Routine?*** A word problem is read three times, with a different question posed with each read: (1) What is this situation about?; (2) What can be counted or measured in this situation?; (3) How might we approach this problem, or what is the first thing you will do to get started? ***Why This Routine?*** *Three Reads* (MLR6) gives students a chance to use everyday language to help each other make sense of the context—and the language—of a word problem before jumping down a solution path. Use this routine to ensure that students know what they are being asked to do, and to create an opportunity for students to reflect on the general structure of quantitative situations and on the ways mathematical questions are presented. This routine supports reading comprehension of problems and meta-awareness of mathematical language. It also supports negotiating information in a text with peers through mathematical conversation.

- First Read: Without displaying the task, read the context aloud to the class: "To get food for a family reunion, Clare went to the local grocery store where she can buy any quantity of a product and the prices are usually good. Clare purchased chicken tenders at $6 a pound and potato wedges at $9 per pound. She spent $75 before tax."
    - Ask students: "What is this situation about? What is going on here?"
    - Let students know the focus is just on the situation, not on the numbers. (For example, students might say, "it's about chicken tenders and potato wedges" or "it's about someone buying food for a reunion and something about how much food and how much it cost").
    - Spend less than a minute scribing their understanding of the situation in a place where all can see. Do not correct students, but do clarify any unfamiliar words; visuals often help (for example, a picture of the deli section of a grocery store).
- Second Read: Display the task context without the problems, and ask a student volunteer to read it aloud to the class again.
    - Ask: "What are the quantities in this situation? A quantity is something that can be counted or measured."
    - Spend less than a minute scribing student responses.

- Encourage students to identify quantities that are named in the problem explicitly and any quantities that may be implicit. For each quantity (for example, “75”), ask students to add details (for example, “the total amount she spent was $75”).

– Third Read: Invite students to read the task context and problems 1–3 to themselves, or ask another student volunteer to read these aloud.

  - Ask: “How might we approach the questions being asked? What is the first thing you will do?”
  - Spend 1–2 minutes scribing student ideas as they brainstorm possible starting points.
  - Be sure to stop any students who begin to share a complete solution; the goal is to crowdsource some starting points.

Step 2

- Provide students 1–2 minutes of quiet work time on questions 1 and 2 before comparing responses with their partner.

Monitoring Tip: Look for students who perform numerical computations straightaway and those who first write a variable equation and then use it to answer the first two questions.

- Have students continue to work together to complete questions 3 and 4.

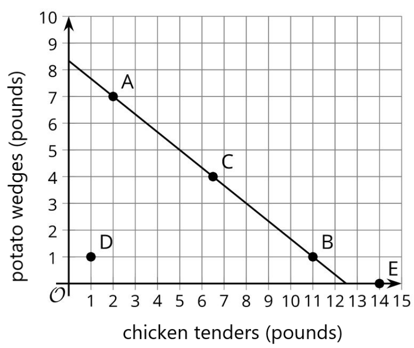
Advancing Student Thinking: Some students may say that the points not on the line are impossible given that Clare spent $75. Encourage these students to think about what those points would mean if we didn’t know how much money Clare spent.

Student Task Statement

To get food for a family reunion, Clare went to the local grocery store where she can buy any quantity of a product and the prices are usually good.

Clare purchased chicken tenders at $6 per pound and potato wedges at $9 per pound. She spent $75 before tax.

1. If she bought 2 pounds of chicken tenders, how many pounds of potato wedges did she buy?
2. If she bought 1 pound of potato wedges, how many pounds of chicken tenders did she buy?
3. Write an equation that describes the relationship between pounds of potato wedges and pounds of chicken tenders that Clare bought and the dollar amount that she paid. Be sure to specify what the variables represent.
4. Here is a graph that represents the quantities in this situation.
   a. Choose any labeled point on the line, state its coordinates, and explain what it tells us.
   b. Choose any labeled point that is *not* on the line, state its coordinates, and explain what it tells us.



Step 3

- Ask previously selected students to share how they solved for questions 1 and 2, highlighting both students who performed numerical calculations and those who used an equation to solve. Show the connections between these different methods.
- Display the graph for all to see. Invite students to share their equation for the situation and their interpretations of the points on and off the graph. Make sure students understand that a point on the graph of an equation in two variables is a solution to the equation.
- Facilitate a whole-class discussion by asking students questions such as:
  - "What does the point $(10, 3)$ mean in this situation?" (Clare purchased 10 pounds of chicken tenders and 3 pounds of potato wedges.)
  - "Is that a possible combination of pounds of potato wedges and chicken tenders? Why or why not?" (No. It doesn't lie on the graph. Also, if Clare bought 10 pounds of chicken tenders and 3 pounds of potato wedges, it would cost her $87, not $75.)
  - "From the graph, it looks like $(7, 3.5)$ might be a solution, but it is hard to know for sure. Is there a way to verify?" (Substitute the values into the equation and see if they make the equation true.)
  - "Suppose we extend the two ends of the graph beyond the first quadrant. Would a point on those parts of the line—say, $(-1, 9)$—be a solution to the equation $6c + 9p = 75$? Why or why not?" (It would still be a solution to the equation, but it wouldn't make sense in this context. The weight of chicken tenders or potato wedges cannot be negative.)

RESPONSIVE STRATEGY

Alternatively, allow groups to develop a collaborative poster. Provide each student with a colored marker to encourage each student to participate. Have students report out to the class. Lastly, follow with a Gallery Walk where students can add their thoughts to other posters.

DO THE MATH	PLANNING NOTES

Activity 2: Graph It! (*15 minutes*)

Instructional Routines: Graph It; Discussion Supports (MLR8) - Responsive Strategy
Addressing: NC.M1.A-CED.2; NC.M1.A-REI.10

In the previous activity, students analyzed and interpreted points on a graph relative to an equation and a situation. In this activity, they write a linear equation to model a situation, use Desmos to graph the equation, and then use the graph to solve problems. Each given situation involves an initial value and a constant rate of change.

This is the first time in the course that students will participate in a *Graph It* routine.

GRAPH IT

What Is This Routine? *Graph It* indicates activities where students have an opportunity to use graphing technology to visualize a graph representing one or more functions with known parameters and use the tool to find features like intersection points, intercepts, and maximums or minimums. Additionally, they may use sliders for exploring the effect of changing parameters.

Why This Routine? Using graphs to solve problems can be cognitively demanding for students not yet fluent with creating this type of representation. Through accessing technology to assist with exploring the nature of graphs, the *Graph It* routine provides students a scaffold for interpreting the structure of the coordinate plane and developing fluency in creating graphs on their own.

Step 1

- Provide students with a computer and access to Desmos. Explain that, in this course, they will frequently use technology to create a graph that represents a relationship and use the graph to solve problems.
- Give students a few minutes to answer the first three questions (through writing an equation). Then have them graph the equation, adjust the viewing window, and answer the last question.

Advancing Student Thinking: If students need assistance adjusting the window, remind them to click on the wrench icon in the top right of Desmos and change values under x-axis and y-axis. They can also change the intervals along the axes by adjusting the step number. For this activity it may be helpful to have students reason about the y-intercept and slope in determining an appropriate window.

Student Task Statement

A student has a savings account with $475 in it. She deposits $125 of her paycheck into the account every week.

1. How much will be in the account after 3 weeks?
2. How long will it take before she has $1,350?
3. Write an equation that represents the relationship between the dollar amount in her account and the number of weeks of saving.
4. Graph your equation using graphing technology. Note the points on the graph that represent the amount after 3 weeks and the week she has $1,350. Write down the coordinates.
5. She determines her goal is to save $7,000 for college. How long will it take her to reach her goal?

Are You Ready For More?

1. A 450-gallon tank full of water is draining at a rate of 20 gallons per minute.
   a. How many gallons will be in the tank after 7 minutes?
   b. How long will it take for the tank to have 200 gallons?
   c. Write an equation that represents the relationship between the gallons of water in the tank and minutes the tank has been draining.
   d. Graph your equation using graphing technology. Mark the points on the graph that represent the gallons after 7 minutes and the time when the tank has 200 gallons. Write down the coordinates.
   e. How long will it take until the tank is empty?
2. Write an equation that represents the relationship between the gallons of water in the tank and *hours* the tank has been draining.
3. Write an equation that represents the relationship between the gallons of water in the tank and *seconds* the tank has been draining.
4. Graph each of your new equations. In what way are all of the graphs the same? In what way are they all different?
5. How would these graphs change if we used quarts of water instead of gallons? What would stay the same?

Step 2

- Ask some students to display their graphs for all to see.
- Facilitate a whole-class discussion on two things: the meanings of the points on the graph and how the graph could be used to answer questions about the quantities in each situation. Discuss questions such as:
  - "How did you find the answers to the first two questions?" (By calculation: for example, computing $475 + 125(3)$, or finding $1{,}350 – 475$, and then dividing by 125.)
  - "How did you find the answer to the last question?" (By calculation: for instance, finding $7{,}000 – 475$, and then dividing by 125. Or, alternatively, by using the graph.)
- Highlight how the graph of the equation could be used to answer the questions. If not already mentioned by students, discuss how the graph of $y = 475 + 125x$ can be used to find the answers to all the questions about the student's savings account.

RESPONSIVE STRATEGY

Provide the following sentence frames for students to use while sharing: "To determine how much will be in the account at 3 weeks, I ____," "To determine how long it will take for the student to save $1,350, I ____," and "The graph helped me see ____."

Discussion Supports (MLR8)

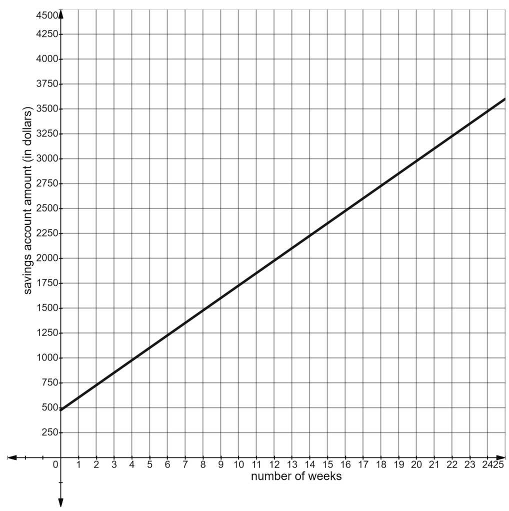
DISCUSSION SUPPORTS	***What Is This Routine?*** The teacher uses multi-modal strategies for helping students comprehend and generate language and ideas, such as sentence frames, word walls, images and videos, revoicing, choral response, gesture, and graphic organizers. The strategies can be combined and used together with any of the other routines. ***Why This Routine?*** *Discussion Supports* (MLR8) foster rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies. Use *Discussion Supports* to make classroom communication accessible, to increase meta-awareness of language, and to demonstrate strategies students can use to enhance their own communication and construction of ideas.

DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is for students to understand that points on the graph of a linear equation are (x, y) pairs that make the equation true. Because of this, graphs can be used to answer questions that might otherwise require algebra to answer. To help students sum up these key ideas, display the graph of the equation from Activity 2.



Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

- “In the activity we just completed, will the student have $5,000 after saving for 20 weeks? How can we use the graph of $y = 475 + 125x$ to answer that question?" (See if the point $(20, 5000)$ is on or below the graph. It is not, so the answer is no.)
- "What does the point $(15, 3000)$ mean?" (After 15 weeks, there is $3,000 in the bank account.)
- "Is that ordered pair a solution to the equation $y = 475 + 125x$? How can we tell?" (No. The point $(15, 3000)$ is above the graph, not on the graph. After 15 weeks, there is less than $3,000 in the account.)
- In general, how can a graph help us find solutions to two-variable equations?" (Any point on the graph of the equation is a solution to that equation.)

PLANNING NOTES

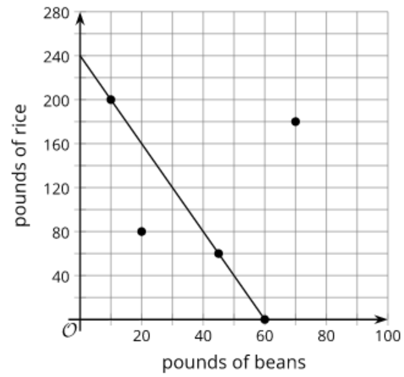
Student Lesson Summary and Glossary

Like an equation, a graph can give us information about the relationship between quantities and the constraints on them.

Suppose we are buying beans and rice to feed a large gathering of people, and we plan to spend $120 on the two ingredients. Beans cost $2 per pound and rice costs $0.50 per pound.

If x represents pounds of beans and y represents pounds of rice, the equation $2x + 0.50y = 120$ can represent the constraints in this situation.

The graph of $2x + 0.50y = 120$ shows a straight line.



Each point on the line is a pair of x- and y-values that makes the equation true and is thus a solution. It is also a pair of values that satisfies the constraints in the situation.

- The point $(10, 200)$ is on the line. If we buy 10 pounds of beans and 200 pounds of rice, the cost will be $2(10) + 0.50(200)$, which equals 120.
- The points $(60, 0)$ and $(45, 60)$ are also on the line. If we buy only beans—60 pounds of them—and no rice, we will spend $120. If we buy 45 pounds of beans and 60 pounds of rice, we will also spend $120.

What about points that are *not* on the line? They are not solutions because they don't satisfy the constraints, but they still have meaning in the situation.

- The point $(20, 80)$ is not on the line. Buying 20 pounds of beans and 80 pounds of rice costs $2(20) + 0.50(80)$ or $80, which does not equal $120. This combination costs less than what we intend to spend.
- The point $(70, 180)$ means that we buy 70 pounds of beans and 180 pounds of rice. It will cost $2(70) + 0.50(180)$ or $230, which is over our budget of $120.

Cool-down: A Spoonful of Sugar (*5 minutes*)

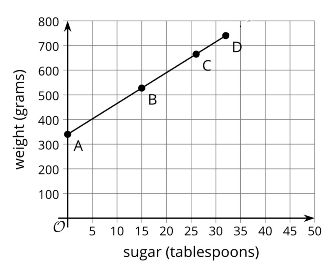
Addressing: NC.M1.A-CED.2; NC.M1.A-REI.10

Cool-down Guidance: Points to Emphasize
Spend 5 minutes at the beginning of the next class reviewing the cool-down with students who struggled with this and try practice problem 4 for an additional opportunity to practice.

Cool-down

A ceramic sugar bowl weighs 340 grams when empty. It is then filled with sugar. One tablespoon of sugar weighs 12.5 grams.

1. Write an equation to represent the relationship between the total weight of the bowl in grams, W, and the tablespoons of sugar, T.
2. When the sugar bowl is full, it weighs 740 grams. How many tablespoons of sugar can the bowl hold? Show your reasoning.
3. The graph represents the relationship between the number of tablespoons of sugar in the bowl and the total weight of the bowl. Which point on the graph could represent your answer to the previous question?
4. About how many tablespoons of sugar are in the bowl when the total weight is 600 grams?



Student Reflection:

When given the chance to write an equation to represent a real-world situation, what is easiest for you and what, if anything, makes it difficult?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

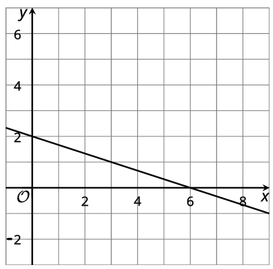
Think about which students haven't shared their strategies in class lately. Were there missed opportunities to highlight their thinking during recent lessons? How can you take advantage of those opportunities when they arise?

Practice Problems

1. Select **all** the points that are on the graph of the equation $4y - 6x = 12$.

   a. $(-4, -3)$
   b. $(-1, 1.5)$
   c. $(0, -2)$
   d. $(0, 3)$
   e. $(3, -4)$
   f. $(6, 4)$

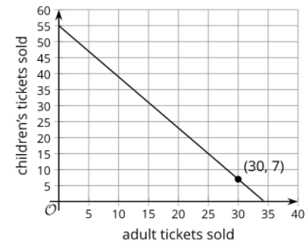
2. Here is a graph of the equation $x + 3y = 6$. Select **all** coordinate pairs that represent a solution to the equation.



   a. $(0, 2)$
   b. $(0, 6)$
   c. $(2, 6)$
   d. $(3, 1)$
   e. $(4, 1)$
   f. $(6, 2)$

3. A theater is selling tickets to a play. Adult tickets cost $8 each, and children's tickets cost $5 each. They collect $275 after selling x adult tickets and y children's tickets.

   What does the point $(30, 7)$ mean in this situation?



4. (*Technology required.*) Priya starts with $50 in her bank account. She then deposits $20 each week for 12 weeks.

   a. Write an equation that represents the relationship between the dollar amount in her bank account and the number of weeks of saving.

   b. Graph your equation using Desmos or other graphing technology. Mark the point on the graph that represents the amount after 3 weeks.

   c. How many weeks does it take her to have $250 in her bank account? Mark this point on the graph.

5. A student on the cross-country team runs 30 minutes a day as a part of her training.

   Write an equation to describe the relationship between the distance she runs in miles, D, and her running speed, in miles per hour, when she runs:

   a. at a constant speed of 4 miles per hour for the entire 30 minutes

   b. at a constant speed of 5 miles per hour the first 20 minutes, and then at 4 miles per hour the last 10 minutes

   c. at a constant speed of 6 miles per hour the first 15 minutes, and then at 5.5 miles per hour for the remaining 15 minutes

   d. at a constant speed of a miles per hour the first 6 minutes, and then at 6.5 miles per hour for the remaining 24 minutes

   e. at a constant speed of 5.4 miles per hour for m minutes, and then at b miles per hour for n minutes

   (From Unit 2)

6. In the 21st century, people measure length in feet and meters. At various points in history, people measured length in hands, cubits, and paces. There are 9 hands in 2 cubits. There are 5 cubits in 3 paces.

   a. Write an equation to express the relationship between hands, h, and cubits, c.

   b. Write an equation to express the relationship between hands, h, and paces, p.

   (From Unit 2)

7. The table shows the amount of money, A, in a savings account after m months.

   Select **all** the equations that represent the relationship between the amount of money, A, and the number of months, m.

   a. $A = 100m$
   b. $A = 100(m–5)$
   c. $A–700 = 100m$
   d. $A–1,200 = 100m$
   e. $A = 700 + 100m$
   f. $A = 1200 + 100m$
   g. $A = 1,200 + 100(m–5)$

Number of months	Dollar amount
5	1,200
6	1,300
7	1,400
8	1,500

(From Unit 2)

8. Solve each equation for y.

   a. $(y-10)=-3(x-2)$
   b. $(y-2)=3(x+1)$
   c. $(y-2)=\frac{1}{3}(x-3)$

   (From Unit 2)

9. During the month of August, the mean of the daily rainfall in one city was 0.04 inches with a standard deviation of 0.15 inches. In another city, the mean of the daily rainfall was 0.01 inches with a standard deviation of 0.05 inches.

   What does the given information tell you about the two cities' patterns of precipitation in the month of August? Explain your reasoning.

   (From Unit 1)

10. Solve each equation.[1]

    a. $2(x-3)=14$
    b. $-5(x-1)=40$
    c. $12(x+10)=24$
    d. $\frac{1}{6}(x+6)=11$
    e. $\frac{5}{7}(x-9)=25$

    (Addressing NC.7.EE.4)

[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Lesson 2: Connecting Equations to Graphs (Part One)

PREPARATION

Lesson Goals	Learning Targets
• Analyze how the numbers in an equation $ax + by = c$ are reflected in its graph and are related to the rate of change in the relationship. • Graph linear equations of the form $ax + by = c$ and interpret points on the graph in context. • Understand that different forms of a linear equation can give different insights about the relationship it represents and about the graph.	• I can describe the connections between an equation of the form $ax + by = c$, the features of its graph, and the rate of change in the situation. • I can graph a linear equation of the form $ax + by = c$. • I understand that rewriting the equation for a line in different forms can make it easier to find certain kinds of information about the relationship and about the graph.

Lesson Narrative

Previously, students have written and interpreted equations that model quantitative relationships and constraints. They have also rearranged and solved equations, isolated one of the variables, and explained why the steps taken to rewrite equations are legitimate.

In this lesson, students consider how parts of two-variable linear equations—the parameters and variables—relate to features of the graphs of those equations. They also think about how different forms of two-variable equations affect the information we could gain about the relationships between the quantities and about the graphs. Throughout the lessons, students practice reasoning quantitatively and abstractly (MP2) as they interpret equations and graphs in context.

Which Standards for Mathematical Practice do you anticipate students engaging in during this lesson? How will you support them?

Focus and Coherence

Building On	Addressing
NC.8.F.4: Analyze functions that model linear relationships. • Understand that a linear relationship can be generalized by $y=mx+b$. • Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two (x, y) values or a graph. • Construct a graph of a linear relationship given an equation in slope-intercept form. • Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and y-intercept of its graph or a table of values. **NC.M1.A-SSE.1a:** Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.	**NC.M1.A-CED.2:** Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities. **NC.M1.A-CED.4:** Solve for a quantity of interest in formulas used in science and mathematics using the same reasoning as in solving equations. **NC.M1.A-REI.10:** Understand that the graph of a two variable equation represents the set of all solutions to the equation.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*10 minutes*)
- **Activity 1** (*20 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U3.L2 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

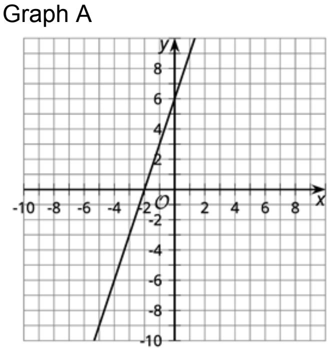
Building On: NC.8.F.4

In this bridge, students practice recognizing key features of graphs of linear relationships and generating equivalent equations. Encourage students to use Desmos to check their thinking. In the lesson after this Bridge, students use situations to build connections with two-variable linear equations and graphs. This task is aligned to question 1 in Check Your Readiness.

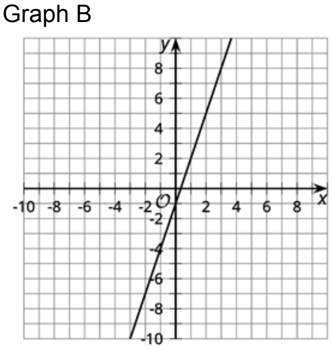
Student Task Statement

Here are the graphs of four equations:

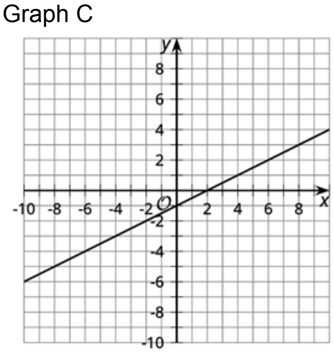
Graph A



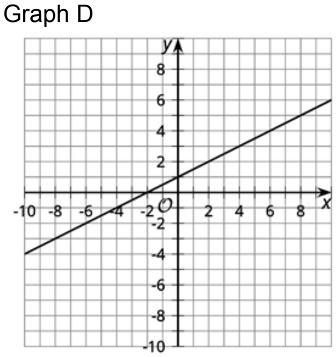
Graph B



Graph C



Graph D



1. Which graphs have a slope of 3?
2. Which graphs have a slope of ½?
3. Which graphs have a y-intercept of -1?
4. Which graphs have an x-intercept of -2?

DO THE MATH

PLANNING NOTES

Warm-up: Games and Rides (*10 minutes*)

Building On: NC.M1.A-SSE.1a

Throughout this lesson, students will use a context that involves two variables—the number of games and the number of rides at a carnival—and a budgetary constraint. This warm-up prompts students to interpret and make sense of some equations in context, familiarizing them with the quantities and relationships (MP2). Later in the lesson, students will dig deeper into what the parameters and graphs of the equations reveal.

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Provide students a couple of minutes of quiet work time and then another minute to share their response with their partner.

Student Task Statement

Jada has $20 to spend on games and rides at a carnival. Games cost $1 each and rides are $2 each.

1. Given this situation, what can Jada do with her $20?
2. Which equation represents the relationship between the number of games, x, and the number of rides, y, that Jada could go on if she spends all her money?
   a. $x + y = 20$
   b. $2x + y = 20$
   c. $x + 2y = 20$
3. Explain what each of the other two equations could mean in this situation.

Step 2

- Invite students to share an interpretation of an equation shared by their partner.
- Most students are likely to associate the 20 in the equation with the $20 that Jada has, but some students may interpret it to mean the combined number of games and rides Jada enjoys. (This is especially natural to do for $x + y = 20$.) If this interpretation comes up, acknowledge that it is valid.

DO THE MATH	PLANNING NOTES

Activity 1: Graphing Games and Rides (*20 minutes*)

Instructional Routines: Collect and Display (MLR2); Round Robin	
Building On: NC.8.F.4	**Addressing:** NC.M1.A-CED.4; NC.M1.A-REI.10

This activity is the first of several that draw students' attention to the structure of linear equations in two variables, how it relates to the graphs of the equations, and what it tells us about the situations.

Students start by interpreting linear equations in standard form, $Ax + By = C$, and using them to answer questions and create graphs. They see that this form offers useful insights about the quantities and constraints being represented. They also notice that graphing equations in this form is fairly straightforward. We can use any two points to graph a line, but the two intercepts of the graph (where one quantity has a value of 0) can be quickly found using an equation in standard form.

Students then analyze the graphs to gain other insights. They determine the rate of change in each relationship and find the slope and vertical intercept of each graph. Next, they rearrange the equations to isolate y. They make new connections here—the rearranged equations are now in slope-intercept form, which shows the slope of the graph and its vertical intercept. These values also tell us about the rate of change and the value of one quantity when the other quantity is 0.

Step 1

- Share with students that they will now interpret some other equations about games and rides. They will also use graphs to help make sense of what combinations of games and rides are possible given certain prices and budget constraints.
- Read the opening paragraph in the task statement and display the three equations for all to see. Give students a minute of quiet time to think about what each equation means in the situation and then invite students to share their meaning with the class.
- Using the *Collect and Display* routine, listen for and collect language students use to describe the meaning of the three equations. Record a written interpretation next to each of the three equations on a visual display. Use arrows or annotations to highlight connections between specific language of the interpretations and the parts of the equations. This will provide students with a resource to draw language from during small-group and whole-group discussions. If students don't offer interpretations similar to those stated below, model this reasoning by providing the first one.

- Equation 1: Games and rides cost $1 each, and the student is spending $20 on them.
- Equation 2: Games cost $2.50 each, and rides cost $1 each. The student is spending $15 on them.
- Equation 3: Games cost $1 each, and rides cost $4 each. The student is spending $28 on them.

***Advancing Student Thinking*:** Some students may not know how to interpret the phrase "for every additional game that a student plays." Suggest to students that they compare how many rides they could take if they played 3 games, to the number of rides they could take if they played 4 games. What about if they played 5 games? Ask them to notice how the number of rides changes when one more game is played.

COLLECT AND DISPLAY	***What Is This Routine?*** The teacher captures students' oral words and phrases into a stable, collective reference in order to stabilize the fleeting language that students use during partner, small-group, or whole-class activities. The teacher listens for, and scribes, the student output using written words, diagrams, and pictures; this collected output can be organized, revoiced, or explicitly connected to other language in a display for all students to use over the course of a lesson or unit. ***Why This Routine?*** *Collect and Display* (MLR2) provides feedback for students in a way that increases accessibility while simultaneously supporting meta-awareness of language. The routine mirrors student language back to the whole class to enable students' own output to be used as a reference in developing their mathematical language over time.

Step 2

- Ask students to arrange themselves in small groups or use visibly random grouping.
- Ask each group to choose an equation and answer the questions for that equation.
  - Give students 2–3 minutes of quiet work time for questions 1–4 only, letting them know they will share their individual responses with their groups following the *Round Robin* routine. Then tell groups to spend a few minutes to share and discuss their responses with their group, making sure each group member shares all of their responses before discussion, and resolve any disagreements. After reaching consensus for questions 1–4, groups should collaborate to complete questions 5–7.

Monitoring Tip: As students work, monitor and listen closely to group discussions to select responses to share with the whole class afterwards in Step 3 and the Lesson Debrief.

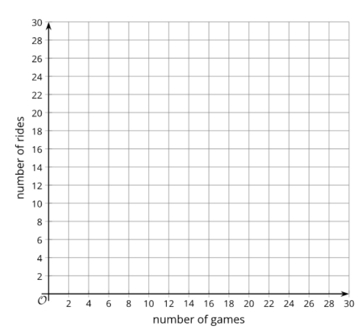
- Ask groups that finish early to answer the questions for a second equation of their choice.

Student Task Statement

Here are the three equations. Each represents the relationship between the number of games, x, the number of rides, y, and the dollar amount a student is spending on games and rides at a different carnival.

- Equation 1: $x + y = 20$
- Equation 2: $2.50x + y = 15$
- Equation 3: $x + 4y = 28$

As a group, choose one of the equations above and answer the questions below based on that equation.



Equation ______________:

1. What's the number of rides the student could get on if they don't play any games? On the coordinate plane, mark the point that represents this situation and label the point with its coordinates.
2. What's the number of games the student could play if they don't get on any rides? On the coordinate plane, mark the point that represents this situation and label the point with its coordinates.
3. Draw a line to connect the two points you've drawn.
4. Complete the sentences: "If the student played no games, they can get on ________ rides. For every additional game that the student plays, x, the possible number of rides, y, ___________ (increases or decreases) by ______________."
5. What is the slope of your graph? Where does the graph intersect the vertical axis?
6. Rearrange the equation to solve for y.
7. What connections, if any, do you notice between your new equation and the graph?

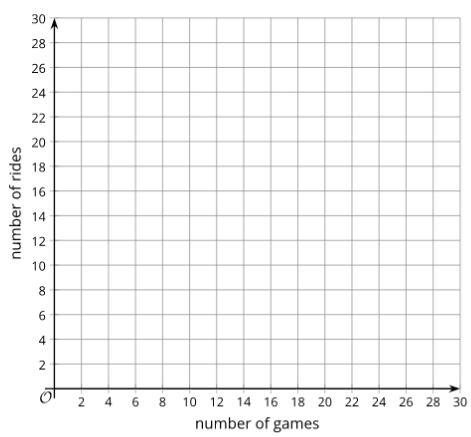
Are You Ready For More?

Here are the same three equations as the task statement above. Each represents the relationship between the number of games, x, the number of rides, y, and the dollar amount a student is spending on games and rides at a different carnival.

- Equation 1: $x + y = 20$
- Equation 2: $2.50x + y = 15$
- Equation 3: $x + 4y = 28$

Choose a different equation above and answer the questions below based on that equation.

Equation ______________:



1. What's the number of rides the student could get on if they don't play any games? On the coordinate plane, mark the point that represents this situation and label the point with its coordinates.
2. What's the number of games the student could play if they don't get on any rides? On the coordinate plane, mark the point that represents this situation and label the point with its coordinates.
3. Draw a line to connect the two points you've drawn.
4. Complete the sentences: "If the student played no games, they can get on ________ rides. For every additional game that the student plays, x, the possible number of rides, y, ___________ (increases or decreases) by ______________."
5. What is the slope of your graph? Where does the graph intersect the vertical axis?
6. Rearrange the equation to solve for y.
7. What connections, if any, do you notice between your new equation and the graph?

Step 3

- Select students to briefly share the graphs and responses. Keep the original equations, the rearranged equations, and their graphs displayed for all to see during discussion.
- To help students see the connections between linear equations in standard form and their graphs, refer to any collected and displayed student language, and ask students:
  - "How did you find the number of possible rides when the student plays no games?" (Substitute 0 for x and solve for y.)
  - "How did you find the number of possible games when the student gets on no rides?" (Substitute 0 for y and solve for x.)
  - "Where on the graph do we see those two situations (all games and no rides, or all rides and no games)?" (on the vertical and horizontal axes or the y- and x-intercepts.)
  - Given the context of games and rides, do all the points on the line make sense? (No. The point $(8.5,\ 11.5)$ is on the graph of $x+y=20$ but you can't play 8 ½ games or go on 11 ½ rides.)
- To help students see that an equivalent equation in slope-intercept form reveals other insights about the situation and the graph, discuss:
  - "If we rearrange the first equation and solve for y, we get the equation $y=20-x$. Is the graph of this equation different from that of the original equation?" (No, the equations are equivalent, so they have the same graph.)
  - "You were asked to complete some sentences about what would happen if the student played more games. How did the graph help you complete the sentences?" (The graph shows how many rides the student can get on if they played no games. The line slants downward, which means that the more games are played, the fewer rides are possible. The graph shows how much the y-value (number of rides) drops when the x-value (number of games) goes up by 1.)
  - "Would you have been able to see the trade-offs between games and rides by looking at the original equations in standard form?" (No, not easily.)

RESPONSIVE STRATEGY

Demonstrate, and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, use the same color to illustrate where the slope appears in each equation and corresponding graph. Continue to use colors consistently as students discuss "What do the A, B, and C represent in each equation?"

Supports accessibility for: Visual-spatial processing

DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson was to highlight that each form of an equation gives us some insights about the relationship between the quantities. Solving for y reveals the slope and y-intercept, which are handy for creating or visualizing a graph. Even without a graph, the slope and y-intercept can communicate the relationship between the quantities.

To help students consolidate their work in this lesson, discuss questions such as:

- "We saw equations in different forms representing the same constraint. For example, $x+4y=28$ and $y=-\frac{1}{4}x+7$ both represent the games and rides that a student could go on with a fixed budget. What information about the situation and about the graph can we gain from the standard form, $Ax+By=C$?" (In this example, the standard form allows us to see the cost per ride, the cost per game, and the budget.)
- "What information does the slope-intercept form give us?" (It gives us the slope and y-intercept of the graph. The slope tells us what is given up in terms of rides for each additional game played. The y-intercept tells us how many rides are possible when no games are played.)
- "What might be an efficient way to graph an equation of the form $Ax+By=C$?" (Substituting 0 for x or for y in the equation. Doing so gives us $(x,0)$ and $(0,y)$, which are the horizontal and vertical intercepts of the graph. We could choose two other points, as well, but using 0 eliminates one of the variables, simplifying the calculation. Alternatively, we could isolate y and rearrange the equation into slope-intercept form, which shows us the y-intercept and the slope.)

PLANNING NOTES

Student Lesson Summary and Glossary

Linear equations can be written in different forms. Some forms allow us to better see the relationship between quantities or to find features of the graph of the equation.

Suppose an athlete wishes to burn 700 calories a day by running and swimming. He burns 17.5 calories per minute of running and 12.5 calories per minute of freestyle swimming.

Let x represent the number of minutes of running and y the number of minutes of swimming. To represent the combination of running and swimming that would allow him to burn 700 calories, we can write:

$$17.5x+12.5y=700$$

We can reason that the more minutes he runs, the fewer minutes he has to swim to meet his goal. In other words, as x increases, y decreases. If we graph the equation, the line will slant down from left to right.

If the athlete only runs and doesn't swim, how many minutes would he need to run?

Let's substitute 0 for y to find x:

$$\begin{aligned}17.5x+12.5(0)&=700\\17.5x&=700\\x&=\frac{700}{17.5}\\x&=40\end{aligned}$$

On a graph, this combination of times is the point $(40,0)$, which is the x-intercept.

If he only swims and doesn't run, how many minutes would he need to swim?

Let's substitute 0 for x to find y:

$$\begin{aligned} 17.5(0) + 12.5y &= 700 \\ 12.5y &= 700 \\ y &= \frac{700}{12.5} \\ y &= 56 \end{aligned}$$

On a graph, this combination of times is the point $(0, 56)$, which is the y-intercept.

If the athlete wants to know how many minutes he would need to swim if he runs for 15 minutes, 20 minutes, or 30 minutes, he can substitute each of these values for x in the equation and find y. Or, he can first solve the equation for y:

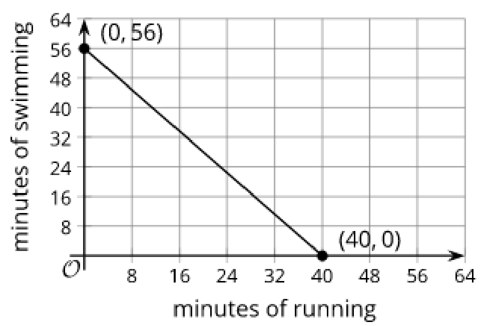
$$\begin{aligned} 27.5x + 12.5y &= 700 \\ 12.5y &= 700 - 17.5x \\ y &= \frac{700 - 17.5x}{12.5} \\ y &= 56 - 1.4x \end{aligned}$$

Notice that $y = 56 - 1.4x$, or $y = -1.4x + 56$, is written in slope-intercept form.

- The coefficient of x, -1.4, is the slope of the graph. It means that as x increases by 1, y falls by 1.4. For every additional minute of running, the athlete can swim 1.4 fewer minutes.
- The constant term, 56, tells us where the graph intersects the y-axis. It tells us the number of minutes the athlete would need to swim if he does no running.

The first equation we wrote, $17.5x + 12.5y = 700$, is a linear equation in standard form. In general, it is expressed as $Ax + By = C$, where x and y are variables, and A, B, and C are numbers.

The two equations, $17.5x + 12.5y = 700$ and $y = -1.4x + 56$, are equivalent, so they have the same solutions and the same graph.



Cool-down: Kiran at the Carnival (*5 minutes*)

Addressing: NC.M1.A-CED.2; NC.M1.A-REI.10

Cool-down Guidance: Points to Emphasize

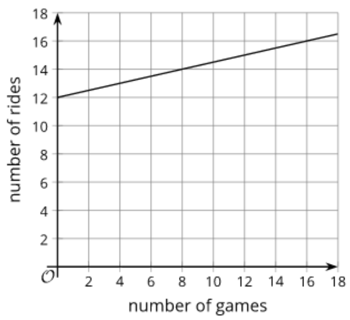
- If students struggle to write the equation (and have struggled writing equations in general), spend 5 minutes in Lesson 3 looking at the kinds of situations that invite equations in slope-intercept form and the kinds of situations that invite an equation in standard form.
- If students struggle to connect the graph to the equation, highlight during Lesson 3 the usefulness of transforming an equation from standard form to slope intercept form in order to find the slope and vertical intercept.

Cool-down

Kiran is spending \$12 on games and rides at another carnival, where a game costs \$0.25 and a ride costs \$1.

1. Write an equation to represent the relationship between the dollar amount Kiran is spending and the number of games, x, he could play and the number of rides, y, he could go on.
2. Which graph represents the relationship between the quantities in this situation? Explain how you know.

a.



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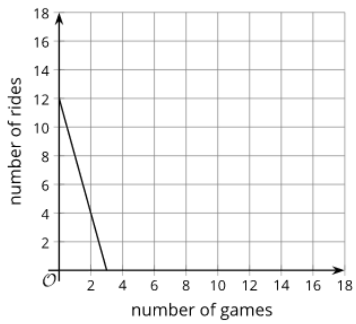

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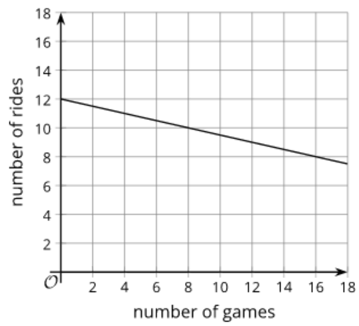


Student Reflection:

After today's lesson, how confident do you feel interpreting the slope and the x- and y-intercepts of a graph? Feel free to share details on how you are feeling.

a. Very confident b. Somewhat confident c. Not confident

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

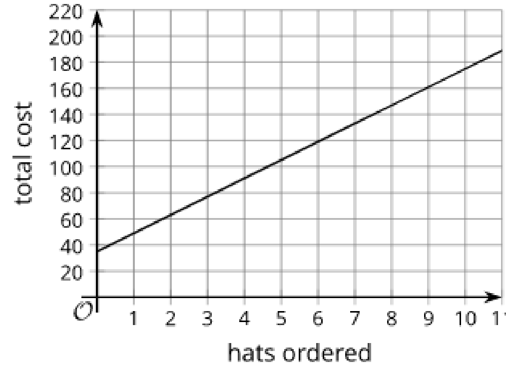
TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

With which math ideas from today's lesson did students grapple most? Did this surprise you or was this what you expected?

Practice Problems

1. A little league baseball team is ordering hats. The graph shows the relationship between the total cost, in dollars, and the number of hats ordered.



What does the **slope** of the graph tell us in this situation?

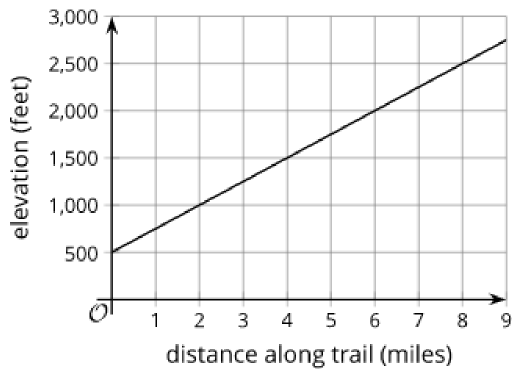
   a. It tells us that there is a fixed cost of approximately \$35 for ordering hats.

   b. It tells us the amount that the total cost increases for each additional hat ordered.

   c. It tells us that when 9 hats are ordered, the total cost is approximately \$160.

   d. It tells us that when the number of hats ordered increases by 10, the total cost increases by approximately \$175.

2. A group of hikers is progressing steadily along an uphill trail. The graph shows their elevation (or height above sea level), in feet, at each distance from the start of the trail, in miles.



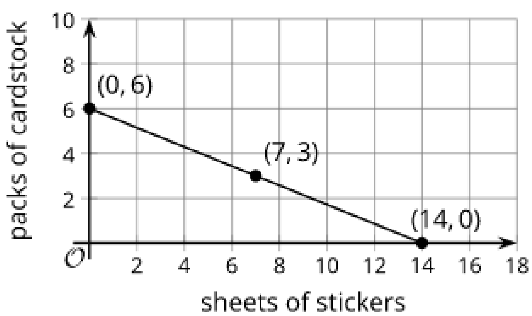
   a. What is the slope of the graph? Show your reasoning.

   b. What does the slope tell us about this situation?

   c. Write an equation that represents the relationship between the hikers' distance from the start of the trail, x, and their elevation, y.

   d. Does the equation $y - 250x = 500$ represent the same relationship between the distance from the start of the trail and the elevation? Explain your reasoning.

3. A kindergarten teacher bought \$21 worth of stickers and cardstock for his class. The stickers cost \$1.50 a sheet, and the cardstock cost \$3.50 per pack. The equation $1.5s + 3.5c = 21$ represents the relationship between sheets of stickers, s, packs of cardstock, c, and the dollar amount the kindergarten teacher spent on these supplies.



   a. Explain how we can tell that this graph represents the given equation.

   b. What do the vertical and horizontal intercepts, $(0,6)$ and $(14,0)$, mean in this situation?

4. Andre bought a new bag of cat food. The next day, he opened it to feed his cat. The graph shows how many ounces were left in the bag on the days after it was bought.

   a. How many ounces of food were in the bag 12 days after Andre bought it?

   b. How many days did it take for the bag to contain 16 ounces of food?

   c. How much did the bag weigh before it was opened?

   d. About how many days did it take for the bag to be empty?

(From Unit 3, Lesson 1)

5. In physics, the equation $PV = nRT$ is called the ideal gas law. It is used to approximate the behavior of many gases under different conditions.

   P, V, and T represent pressure, volume, and temperature, n represents the number of moles of gas, and R is a constant for the ideal gas.

   Which equation is solved for T?

   a. $\frac{PV}{R} = nT$
   b. $\frac{PV}{nR} = T$
   c. $T = PV - nR$
   d. $PVnR = T$

   (From Unit 2)

6. To raise funds for uniforms and travel expenses, the soccer team is holding a car wash in a part of town with a lot of car and truck traffic. The team spent $90 on supplies like sponges and soap. They plan to charge $10 per car and $20 per truck. Their goal is to raise $460.

   How many cars do they have to wash if they washed the following numbers of trucks?

   a. 4 trucks
   b. 15 trucks
   c. 21 trucks
   d. 27 trucks
   e. t trucks

   (From Unit 2)

7. During the Middle Ages, people often used grains, scruples, and drahms to measure the weights of different medicines.

   If 120 grains are equivalent to 6 scruples and 6 scruples are equivalent to 2 drahms, how many drahms are equivalent to 300 grains? Explain your reasoning. If you get stuck, try creating a table.

   (From Unit 2)

8. Explain why the equation $2(3x – 5) = 6x + 8$ has no solutions.

   (From Unit 2)

9. Consider the equation $3a + 0.1n = 123$. If we solve this equation for n, which equation would result?

   a. $0.1n = 123 - 3a$
   b. $n = 123 - 3a - 0.1$
   c. $n = 1,230 - 30a$
   d. $\frac{3a - 123}{0.1} = n$

   (From Unit 2)

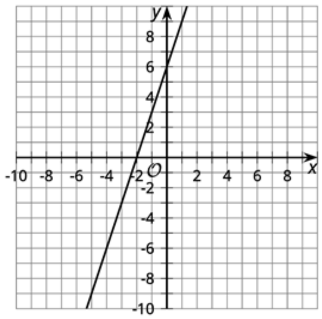
10. Diego is buying shrimp and rice to make dinner. Shrimp costs $6.20 per pound and rice costs $1.25 per pound. Diego spent $10.55 buying shrimp and rice. The relationship between pounds of shrimp s, pounds of rice r, and the total cost is represented by the equation $6.20s + 1.25r = 10.55$.

    Write an equation that makes it easy to find the number of pounds of rice if we know the number of pounds of shrimp purchased.

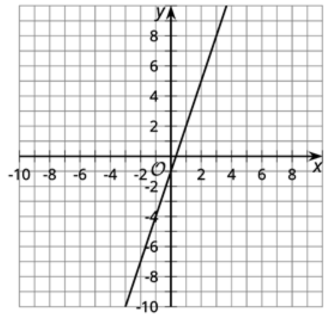
    (From Unit 2)

Use the following graphs to answer questions 11 and 12.

Graph A



Graph B



11. Graph A represents the equation $2y - 6x = 12$. Which other equations could graph A represent?

a. $y - 3x = 6$
b. $y = 3x + 6$
c. $y =- 3x + 6$
d. $2y =- 6x + 12$
e. $4y - 12x = 12$
f. $4y - 12x = 24$

(Addressing NC.8.F.4)

12. Write three equations that graph B could represent.

a. _______________ b. ______________________ c. __________________

(Addressing NC.8.F.4)

Lesson 3: Connecting Equations to Graphs (Part Two)

PREPARATION

Lesson Goals	Learning Targets
• Determine the slope and vertical intercept of the graphs of linear equations by making use of structure or by rearranging the equations. • Given an equation of the form $ax + by = c$, write an equivalent equation of the form $y = mx + b$.	• I can use a variety of strategies to find the slope and vertical intercept of the graph of a linear equation given in different forms. • I can take an equation of the form $ax + by = c$ and rearrange it into the equivalent form $y = mx + b$.

Lesson Narrative

In this lesson, students continue to practice relating the structure of equations to the situation and corresponding graphs. They use their understanding of constraints, equations, and points on a graph to explain whether a graph represents an equation and a situation. Along the way, students practice reasoning quantitatively and abstractly (MP2) and constructing logical arguments (MP3).

Students also work with the structure of linear equations outside of contextual situations. They analyze and rearrange equations to determine the slope and y-intercept of their graphs, and they practice explaining their reasoning.

Technology isn't required for this lesson, but it is recommended to make Desmos available so students can have the option to choose and use appropriate tools strategically (MP5).

Share some ways you see this lesson connecting to previous lessons in this unit. What connections will you want to make explicit?

Focus and Coherence

Building On	Addressing
NC.6.EE.3: Apply the properties of operations to generate equivalent expressions without exponents. **NC.8.F.4:** Analyze functions that model linear relationships. ● Understand that a linear relationship can be generalized by $y=mx+b$. ● Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two (x, y) values or a graph. ● Construct a graph of a linear relationship given an equation in slope-intercept form. ● Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and y-intercept of its graph or a table of values.	**NC.M1.A-CED.2:** Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities. **NC.M1.A-CED.4:** Solve for a quantity of interest in formulas used in science and mathematics using the same reasoning as in solving equations. **NC.M1.A-REI.10:** Understand that the graph of a two variable equation represents the set of all solutions to the equation.

Agenda, Materials, and Preparation

- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*15 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U3.L3 Cool-down (print 1 copy per student)

LESSON

Warm-up: Rewrite These! (*5 minutes*)

Instructional Routine: Discussion Supports (MLR8) - Responsive Strategy	
Building On: NC.6.EE.3	

In this warm-up, students review how to apply the distributive property to rewrite expressions that involve division, preparing them to do so in the next activity in the lesson.

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Give students a moment of quiet time to work on the first two questions and then time to discuss their responses with their partner before moving on the last two questions.

RESPONSIVE STRATEGIES

Include a word bank that will allow students to speak about their thought processes/reasoning. Provide sentence frames for students to use while they are sharing, such as "I decided to rewrite equation 1 as a difference, because...," "In order to keep the numbers positive, I decided to...," and "To change the numbers from positive to negative (or negative to positive) I determined that I would need to..."

Discussion Supports (MLR8)

***Advancing Student Thinking*:** Expect some students to give $2x - 10$ or $4x - 5$ as an answer to the first question. To illustrate why these are incorrect, take an example like $\frac{4+6}{2}$. Explain that we know that 10 divided by 2 is 5, but if we divide only the 4 or only the 6 by 2, we won't get 5. Alternatively, remind students that fraction bars can be interpreted as division, so each expression can be rewritten as, say, $(4x - 10) \div 2$, and we can apply the distributive property.

The signs of the numbers in the second expression might be a source of confusion. Students might be unsure if the expression should be $-\frac{1}{2} - 25x$, $\frac{1}{2} + 25x$, or another expression. Encourage students to substitute a number into the original expression, then try it in each potential answer. To explain why $-\frac{1}{2} + 25x$ is correct, appeal to the distributive property again.

Student Task Statement

Rewrite each quotient as a sum or a difference.

1. $\frac{4x-10}{2}$
2. $\frac{5(x+10)}{25}$
3. $\frac{1-50x}{-2}$
4. $\frac{-\frac{1}{5}x+5}{2}$

Step 2

- Invite students to share their equivalent expressions and how they reasoned about them. Ask students to display their expressions. Make sure students see that rewriting the expressions as sums or differences involves distributing the division (or applying the distributive property on division).
- If not mentioned in students' explanations, point out each division could be thought of in terms of multiplication. For example, $\frac{4x-10}{2}$ is equivalent to $\frac{1}{2}(4x - 10)$, because dividing by a number (in this case, 2) gives the same result as multiplying by the reciprocal of that number (in this case, $\frac{1}{2}$). Applying the distributive property of multiplication to $\frac{1}{2}(4x - 10)$ enables us to rewrite this product as a difference.

DO THE MATH	PLANNING NOTES

Activity 1: Graphs of Two Equations (*15 minutes*)

Instructional Routines: Discussion Supports (MLR8) - Responsive Strategy; Collect and Display (MLR2)	
Building On: NC.6.EE.3	**Addressing:** NC.M1.A-CED.4; NC.M1.A-REI.10

This activity reinforces the understanding that students began to develop in an earlier lesson about the connections between the structure of two-variable linear equations, their graphs, and the situations they represent.

Students first make observations and construct questions about two graphs that they may have seen in a previous lesson. Sentence frames are provided as a *Discussion Supports* responsive strategy. Students then practice relating the parameters of an equation in slope-intercept form to the features of the graph and interpreting them in terms of the situation (MP2). Next, they practice making a case for how they know that a graph represents an equation given in standard form.

The work in this activity requires students to reason quantitatively and abstractly about the equation and the graph (MP2) and to construct a logical argument (MP3).

Step 1

- Display the two graphs in the task statement for all to see. Tell students the graphs represent two situations they have seen in earlier activities. Give students a minute of quiet think time to make observations for or construct questions about the two graphs.
- Invite students to arrange themselves in pairs or use visibly random grouping.
- Ask them to discuss their observations and questions about the graphs with their partner before moving on to the task.
  - As students work, use the *Collect and Display* routine. As you circulate the room, listen for phrases and words that students use that may indicate rearranging equations. Collect their words and ideas and display them for all to see.

RESPONSIVE STRATEGY

Support students in producing statements when they share their observations. Provide sentence frames for students to use when they are sharing, such as "I noticed that…." or "I wondered about ____ because…."

Discussion Supports (MLR8)

RESPONSIVE STRATEGY

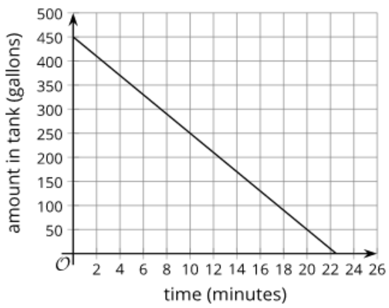
Provide visual aids to support students comprehension of quantities being graphed, such as a picture of a gallon jug or a graph with images on the x- and y-axis.

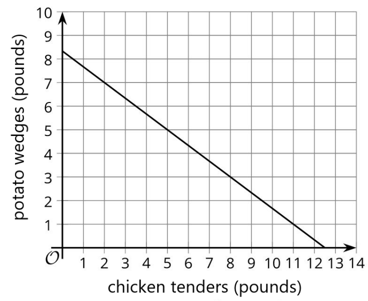
Monitoring Tip: As students work, monitor for the justifications students provide for question 2.

- Some students may argue that substituting the (x, y) pair of any point on the line gives a true statement, suggesting that the graph does match the equation.
- Some students may reason about the points on the graph in terms of chicken fingers and potato wedges and come to the same conclusion. For example, $(8, 3)$ and $(11, 1)$ are points on the line. If Clare buys 8 pounds of chicken tenders and 3 pounds of potato wedges, or 11 pounds of chicken tenders and 1 pound of potato wedges, the price is $75.
- Ask these students how they would check whether the points with fractional x- and y-values (which are harder to identify precisely from the graph) would also produce true statements when those values are substituted. Use this to motivate students into rearranging the equation into slope-intercept form.

Student Task Statement

Here are two graphs that represent situations you may have seen in earlier activities.





1. The first graph represents $a = 450 - 20t$, which describes the relationship between gallons of water in a tank and time in minutes.
    a. Where on the graph can we see the 450? Where can we see the -20?
    b. What do these numbers mean in this situation?
2. The second graph represents $6x + 9y = 75$. It describes the relationship between pounds of chicken tenders and potato wedges and the dollar amount Clare spent on them.

    Suppose a classmate says, "I am not sure the graph represents $6x + 9y = 75$ because I don't see the 6, 9, or 75 on the graph." How would you show your classmate that the graph indeed represents this equation?

Step 2

- Facilitate a discussion focused on students' explanations for the last question, referring to any displayed ideas or phrases that were collected as students worked in pairs. If no one mentions that $6x + 9y = 75$ can be rearranged into an equivalent equation, $y = 8\frac{1}{3} - \frac{2}{3}x$, point this out. (Demonstrate the rearrangement process, if needed.)
- Ask students if we can now see the $8\frac{1}{3}$ and the $-\frac{2}{3}$ on the graph and, if so, where they are visible. To help students connect these values back to the quantities in the situation, ask what each value tells us about chicken tenders and potato wedges. Make sure students see that the $8\frac{1}{3}$ tells us that if Clare bought no chicken tenders, she can buy $8\frac{1}{3}$ pounds of potato wedges. For every pound of chicken tenders she buys, she can buy less of the potato wedges—$\frac{2}{3}$ pound less, to be exact.

DO THE MATH

PLANNING NOTES

Activity 2: Slope Match (*15 minutes*)

Instructional Routines: Graph It; Take Turns; Discussion Supports (MLR8) - Responsive Strategy
Addressing: NC.M1.A-CED.4

Previously, students have studied the structure of equations concretely and contextually. In this *Graph It* activity, they shift to reasoning symbolically and abstractly about linear equations in two variables as they match equations in standard forms to pairs of slopes and y-intercepts. Depending on the approaches students use, they may begin to notice patterns around the structure of the equations with which they are working (MP7 and MP8). Additionally, making Desmos available gives students an opportunity to choose appropriate tools strategically (MP5).

RESPONSIVE STRATEGY

Students should take turns in pairs finding a match and explaining their reasoning to their partner. Display the following sentence frames for all to see: "____ and ____ match because....", and "I noticed ____ , so I matched...." Encourage students to challenge each other when they disagree.

Discussion Supports (MLR8)

Step 1

- Keep students in pairs.
- Ask students to *Take Turns* finding a match and explaining their strategy to their partner.

TAKE TURNS

What Is This Routine? Students work with a partner or small group. They take turns in the work of the activity, whether it be spotting matches, explaining, justifying, agreeing or disagreeing, or asking clarifying questions. If they disagree, they are expected to support their case and listen to their partner's arguments. The first few times students engage in these activities, the teacher should demonstrate, with a partner, how the discussion is expected to go. Once students are familiar with these structures, less set-up will be necessary. While students are working, the teacher can ask students to restate their question more clearly or paraphrase what their partner said.

Why This Routine? Building in an expectation, through the *Take Turns* routine, that students explain the rationale for their choices and listen to another's rationale deepens the understanding that can be achieved through these activities. Specifying that students take turns deciding, explaining, and listening limits the phenomenon where one student takes over and the other does not participate. Taking turns can also give students more opportunities to construct logical arguments and critique others' reasoning (MP3).

Monitoring Tip: As students work, monitor the strategies they use.

- Here are a few likely strategies, from less reliant on structure to more reliant on structure:
  - Substituting the coordinates of the given y-intercepts into each equation and see which one produces true equations. Encourage those who approach it this way to consider rearranging equations or otherwise using the structure of the equations.
  - Calculating the y-values when x is 0 and x is 1. The former gives the y-intercept. Subtracting the latter from the former gives the slope.
  - Graphing each equation (either by hand or using technology) and analyzing the graph.
  - Rearranging the equation into slope-intercept form, $y = mx + b$, and identifying the m and b.
- Some students may notice patterns from manipulating and graphing equations in the past few lessons. For instance, they may observe that when A, B, and C in $Ax + By = C$ are positive, the graph always slants down from left to right and therefore has a negative slope.
- Others may notice that when isolating y in an equation in standard form, the constant term in the resulting equation is $\frac{C}{B}$ and the coefficient of x is $\frac{-A}{B}$, and that these values tell them the vertical intercept and the slope.

Advancing Student Thinking: Students will likely use the strategy of rewriting the equations in slope-intercept form. Common mistakes here include isolating x rather than y, changing the sign of only one term when dividing by a negative number, and dividing only one of two terms by the coefficient of y. (For these last two mistakes, remind students of the work in the warm-up).

Students who recognize that the slope of a line with equation $Ax + By = C$ is $\frac{-A}{B}$ and that the y-intercept is $\frac{C}{B}$ may also write the wrong signs, or get a ratio reversed. Students who use this strategy are likely shortcutting the process of isolating y. Asking them to isolate y for one equation can help them to identify errors.

Student Task Statement

Match each of the equations with the slope m and y-intercept of its graph.

Equation	Slope and y-intercept
1. $-4x + 3y = 3$	a. $m = 3$, $y - int = (0, 1)$
2. $12x - 4y = 8$	b. $m = \frac{4}{3}$, $y - int = (0, 1)$
3. $8x + 2y = 16$	c. $m = \frac{4}{3}$, $y - int = (0, -2)$
4. $-x + \frac{1}{3}y = \frac{1}{3}$	d. $m = -4$, $y - int = (0, 8)$
5. $-4x + 3y = -6$	e. $m = 3$, $y - int = (0, -2)$

Are You Ready For More?

Each equation in Activity 2 is in the form $Ax + By = C$.

1. For each equation in Activity 2, graph the equation and on the same coordinate plane graph the line passing through $(0, 0)$ and (A, B). What is true about each pair of lines?
2. What are the coordinates of the x-intercept and y-intercept in terms of A, B, and C?

Step 2

Select students to present their strategies, beginning with an example of rewriting in the form $y = mx + b$. After this strategy is presented, poll the class for others who approached it the same way and consider discussing questions such as:

- “Is it necessary to rewrite in $y = mx + b$ form to match with the slope and y-intercept or are there other strategies we could use?" (Elicit examples of all other strategies noticed during monitoring, polling for others with the same each time)
- "How would you describe the matching strategy? Was it fairly efficient or laborious? Was it prone to errors?"
- "If the list of slopes and y-intercepts were not available for you to choose from, would you still be able to determine the slope and y-intercept?" (Yes, except those who rely on using the coordinate values of each y-intercept on the list to test the equations.)
- (For students who use graphing technology:) "Would you still use graphs to make the matches if the graphing needed to be done by hand?"
- “Did anyone see a strategy they think they will use in the future instead of the strategy they used in this activity?”

Highlight that it is helpful and efficient to use the structure of an equation to get insights about the properties of its graph. At this stage, it is not essential that students recognize that the slope of an equation of the form $Ax + By = C$ is $\frac{-A}{B}$ and that it crosses the y-axis at $\frac{C}{B}$. Students should, however, recognize that solving for y involves a predictable process and the resulting equation makes the slope and y-intercept visible.

DO THE MATH	**PLANNING NOTES**

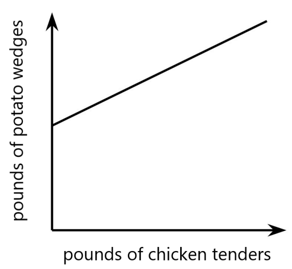
Lesson Debrief (*5 minutes*)

Display the following description and graphs for all to see.

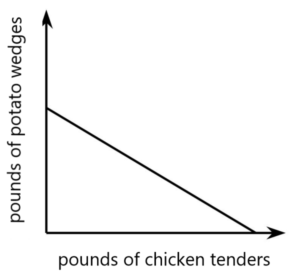
> Suppose Clare went back to the store to get more chicken tenders and potato wedges and spent $108 this time. Chicken tenders cost $6 a pound, and potato wedges cost $9 a pound. Clare's purchase can be represented by the equation $6x + 9y = 108$.

Here are two graphs showing a relationship between pounds of chicken tenders, x, and pounds of potato wedges, y.

Graph A



Graph B



Discuss with students:	**PLANNING NOTES**
• "Without calculating anything, can you tell which graph could represent the equation $6x + 9y = 108$ and which graph could not?" (Yes, graph B could represent the equation. The graph of the equation would have a negative slope because the more chicken tenders Clare bought, the fewer potato wedges she could have afforded. Graph A shows that as more chicken tenders are bought, more potato wedges are also bought, which couldn't be true if Clare spent a fixed amount of money.) • "What does the vertical intercept mean in this situation?" (It shows the pounds of potato wedges Clare could buy if she bought no chicken tenders.) • "What is the vertical intercept of the graph? How can we find it?" (The vertical intercept is $(0, 18)$. One way to find it is to substitute 0 for x and solve for y, which gives $y = 18$. Another way is to rearrange the equation into slope-intercept form.)	

- "What is the slope of the graph? How can we find it?" (The slope is $-\frac{2}{3}$. We can rearrange the equation into slope-intercept form, or we can find the coordinates of another point and calculate the slope.)
- "What does the slope tell us about the chicken tenders and potato wedges?" (It tells us that for every additional pound of chicken tenders that Clare buys, she could buy $\frac{2}{3}$ fewer pounds of potato wedges.)

Student Lesson Summary and Glossary

Here are two situations and two equations that represent them.

Situation 1: Mai receives a $40 bus pass. Each school day, she spends $2.50 to travel to and from school.

Let d be the number of school days since Mai received a pass and b the balance or dollar amount remaining on the pass.

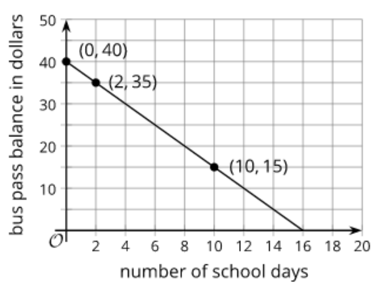
$$b = 40 - 2.50d$$

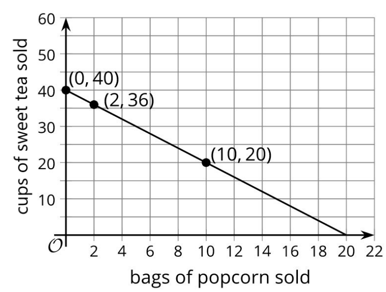
Situation 2: A student club is raising money by selling popcorn and sweet tea. The club is charging $3 per bag of popcorn and $1.50 per cup of sweet tea, and plans to make $60.

Let p be the bags of popcorn sold and t the cups of sweet tea sold.

$$3p + 1.50t = 60$$

Here are graphs of the equations. On each graph, the coordinates of some points are shown.





The 40 in the first equation can be observed on the graph and the -2.50 can be found with a quick calculation. The graph intersects the vertical axis at 40 and the -2.50 is the slope of the line. Every time d increases by 1, b decreases by 2.50. In other words, with each passing school day, the dollar amount in Mai's bus pass drops by 2.50.

The numbers in the second equation are not as apparent on the graph. The values where the line intersects the vertical and horizontal axes, 40 and 20, are not in the equation. We can, however, reason about where they come from.

- If p is 0 (no popcorn is sold), the club would need to sell 40 cups of sweet tea to make $60 because $40(1.50) = 60$.
- If t is 0 (no sweet tea is sold), the club would need to sell 20 bags of popcorn to make $60 because $20(3) = 60$.

What about the slope of the second graph? We can compute it from the graph, but it is not shown in the equation $3p + 1.50t = 60$.

Notice that in the first equation, the variable b was isolated. Let's rewrite the second equation and isolate t:

$$\begin{aligned} 3p + 1.50t &= 60 \\ 1.50t &= 60 - 3p \\ t &= \frac{60-3p}{1.50} \\ t &= 40 - 2p \end{aligned}$$

Now the numbers in the equation can be more easily related to the graph: The 40 is where the graph intersects the vertical axis and the -2 is the slope. The slope tells us that as p increases by 1, t falls by 2. In other words, for every additional bag of popcorn sold, the club can sell 2 fewer cups of sweet tea.

Cool-down: Features of a Graph (*5 minutes*)

Addressing: NC.M1.A-REI.10

Cool-down Guidance: Press Pause
Provide students with more opportunities to practice converting from standard form to slope-intercept form if they continue to struggle here.

Graphing technology should not be used for this cool-down.

Cool-down

Consider the equation $1.5x + 4.5y = 18$. For each question, explain or show your reasoning.

1. If we graph the equation, what is the slope of the graph?
2. Where does the graph intersect the y-axis?
3. Where does it intersect the x-axis?

Student Reflection:

Based on your participation and engagement, please reflect on one of the prompts below:

- I felt confident to engage in the work and participate today because ________________.
- I felt unconfident to engage in the work and participate today because ________________.

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Reflect on how comfortable your students are asking questions of you and of each other. What can you do to encourage students to ask questions?

Practice Problems

1. What is the slope of the graph of $5x - 2y = 20$?
   a. -10
   b. $\frac{-2}{5}$
   c. $\frac{5}{2}$
   d. 5

2. What is the y-intercept of each equation?
   a. $y = 6x + 2$
   b. $10x + 5y = 30$
   c. $y - 6 = 2(3x - 4)$

3. Han wanted to find the intercepts of the graph of the equation $10x + 4y = 20$. He decided to put the equation in slope-intercept form first. Here is his work:

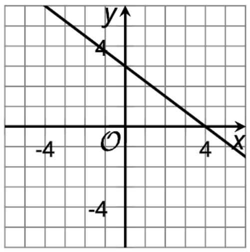
$$10x + 4y = 20$$
$$4y = 20 - 10x$$
$$y = 5 - 10x$$

   He concluded that the x-intercept is $(\frac{1}{2}, 0)$ and the y-intercept is $(0, 5)$.

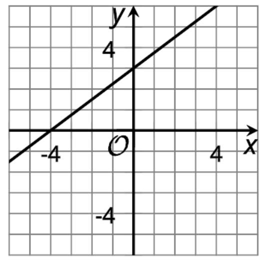
   a. What error did Han make?
   b. What are the x- and y-intercepts of the line? Explain or show your reasoning.

4. Which graph represents the equation $12 = 3x + 4y$? Explain how you know.

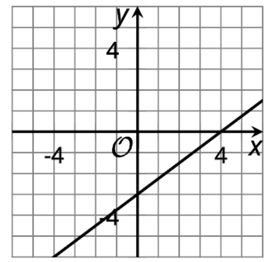
a.



b.



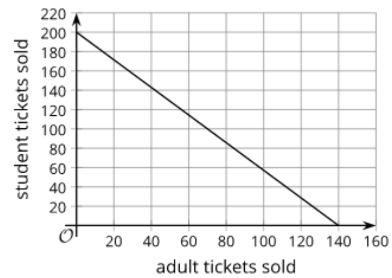
c.



5. A school sells adult tickets and student tickets for a play. It collects $1,400 in total.

The graph shows the possible combinations of the number of adult tickets sold and the number of student tickets sold.

What does the vertical intercept (0, 200) tell us in this situation?



a. It tells us the decrease in the sale of adult tickets for each student ticket sold.
b. It tells us the decrease in the sale of student tickets for each adult ticket sold.
c. It tells us that if no adult tickets were sold, then 200 student tickets were sold.
d. It tells us that if no student tickets were sold, then 200 adult tickets were sold.

(From Unit 3, Lesson 2)

6. Clare knows that Priya has a bunch of nickels and dimes in her pocket and that the total amount is $1.25.

a. Find one possibility for the number of nickels and number of dimes that could be in Priya's pocket.

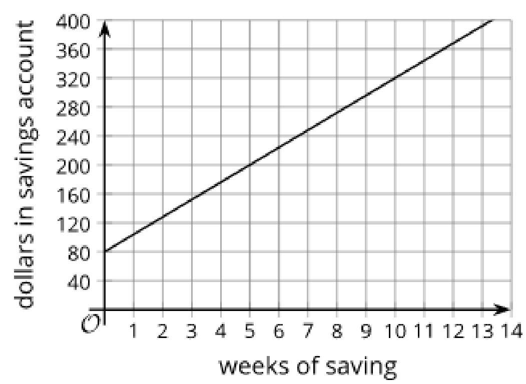
b. Write an equation that describes the relationship between the number of dimes and the number of nickels in Priya's pocket.

c. Explain what the point $(13, 6)$ means in this situation.

d. Is the point $(13, 6)$ a solution to the equation you wrote? Explain your reasoning.

(From Unit 2)

7. The graph shows how much money Priya has in her savings account weeks after she started saving on a regular basis.



a. How much money does Priya have in the account after 10 weeks?

b. How long did it take her to save $200?

c. How much money did Priya have in her savings account when she started to save regularly?

d. Write an equation to represent the dollar amount in her savings account and the number of weeks of saving. Be sure to specify what each variable represents.

(From Unit 2)

8. Noah has a coin jar containing d dimes and q quarters worth a total of $5.00.

   Select **all** the equations that represent this situation.

   a. $d+q=5$
   b. $d+q=500$
   c. $0.1d+0.25q=5$
   d. $10d+25q=500$
   e. $d=50$
   f. $q=20$

   (From Unit 2)

9. Noah orders an extra-large pizza. It costs $\$12.49$ for the pizza plus $\$1.50$ for each topping. He orders an extra-large pizza with t toppings that costs a total of d dollars.

   Select **all** of the equations that represent the relationship between the number of toppings t and total cost d of the pizza with t toppings.

   a. $12.49+t=d$
   b. $12.49+1.50t=d$
   c. $12.49+1.50d=t$
   d. $12.49=d+1.50t$
   e. $t=\frac{d-12.49}{1.5}$
   f. $t=d-\frac{12.49}{1.5}$

   (From Unit 2)

Lesson 4: Equations of Lines

PREPARATION

Lesson Goal	Learning Target
• Generalize (using words and other representations) that a line can be represented by an equation in point-slope form.	• I can use the definition of slope to write the equation for a line in point-slope form.

Lesson Narrative

In grade 8, students used similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane, and they derived the equation $y = mx + b$ for a line intercepting the vertical axis at b. In this lesson, students develop the **point-slope form** of a linear equation: $y - k = m(x - h)$.

As students build towards procedural fluency with writing the equations of lines, it is beneficial to understand and be able to use different forms. Thus far, students have used slope-intercept form and standard form. In this lesson, students derive the point-slope form by rearranging the slope formula (a connection to learning from Unit 2). This form connects to learning in future courses when students study transformations of functions. For now, the focus is on how it connects to slope.

Students will be writing equations of lines in the next several lessons, and intercepts will not always be readily available. Point-slope form will require the least algebraic manipulation and allow students to focus on geometric properties.

Slope calculations are an important part of this lesson, so students begin with a warm-up that helps them recall this concept. Then they use the definition of slope to build the point-slope equation. Finally, they practice writing and interpreting equations of lines in point-slope form. Students have the opportunity to construct a viable argument (MP3) when they explain their methods of calculating slope.

Share some of the benefits of students working with point-slope form.

Focus and Coherence

Building On	Addressing	Building Towards
NC.8.F.4: Analyze functions that model linear relationships. • Understand that a linear relationship can be generalized by $y = mx + b$. • Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two (x, y) values or a graph. • Construct a graph of a linear relationship given an equation in slope-intercept form. • Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and y-intercept of its graph or a table of values.	**NC.M1.A-SSE.1a:** Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.	**NC.M1.G-GPE.5:** Use coordinates to prove the slope criteria for parallel and perpendicular lines and use them to solve problems. • Determine if two lines are parallel, perpendicular, or neither. • Find the equation of a line parallel or perpendicular to a given line that passes through a given point.

Agenda, Materials, and Preparation

- **Warm-up** (*10 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
  - Blank visual displays for each student group (possible visual display options: poster board, chart paper, Google Slides, Jamboard)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U3.L4 Cool-down (print 1 copy per student)

LESSON

Warm-up: Remembering Slope (*10 minutes*)

Building On: NC.8.F.4	**Building Towards:** NC.M1.G-GPE.5

This task emphasizes the form of expressions for slope and reviews the concept of a slope triangle. This work will lead to the development of the point-slope form of a linear equation in the next activity.

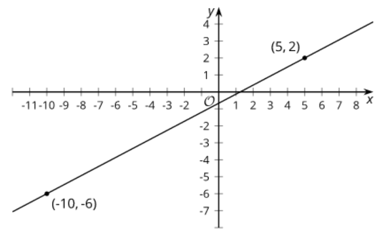
Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping. Tell students that there are many possible answers for the question.
- After quiet work time, ask students to compare their responses to their partner's and decide if they are both correct, even if they are different.

> M **Monitoring Tip:** As students work, monitor for students who draw a slope triangle and for those who use a slope formula. Select a student who drew a slope triangle to share their work in Step 2. For any students that used a slope formula, prepare to have them explain the expressions within the formula in Step 2.

Student Task Statement

The slope of the line in the image is $\frac{8}{15}$. Explain how you know this is true.



Step 2

The goal of the discussion is to highlight the expression $\frac{2-(-6)}{5-(-10)}$. Seeing the coordinates subtracted in this way will help students as they work through upcoming activities.

- Ask students what “slope” means. Students may describe slope as the steepness of the line, the rate of change in a linear relationship, or “rise over run.” Tell students that one way to think about slope is that it is the quotient of the lengths of the legs of a slope triangle: $\textit{vertical distance} \div \textit{horizontal distance}$. A right triangle drawn between any two points on a line will produce the same slope result.
- Invite a student who drew a slope triangle, preferably a student with incomplete understanding but who has a triangle drawn, to share their work. Display the student’s slope triangle for all to see, or draw one of your own. Ask the students how they can calculate the lengths of the legs of the triangle. As students describe how to do so, label the legs $2-(-6)$ and $5-(-10)$.
- Now write out the slope as $\frac{2-(-6)}{5-(-10)}$. It’s important that students see this expression in preparation for their work in the next activity. If any students used a slope formula, ask them how the formula relates to this expression. Ask students if the order of the numbers matters. (The order must be consistent. Because we started with the 2 in the numerator, we have to start with the 5 in the denominator.)

DO THE MATH	PLANNING NOTES

Activity 1: Building an Equation for a Line (*15 minutes*)

Instructional Routine: Discussion Supports (MLR8) - Responsive Strategy	
Addressing: NC.M1.A-SSE.1a	**Building Towards:** NC.M1.G-GPE.5

In this task, students develop the point-slope form of the equation of a line.

Step 1

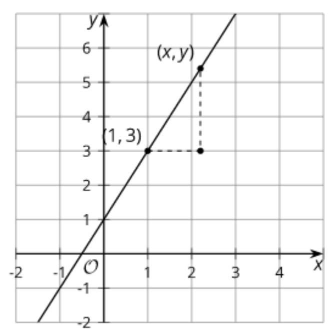
- With students remaining in pairs, provide 5 minutes of quiet independent time for students to work through questions 1 and 2.
- After 5 minutes, ask students to discuss their answers to questions 1 and 2 with their partner.
- When ready to move on, ask students to answer question 3 collaboratively.

Advancing Student Thinking: If students struggle with the first question, suggest they label the lengths of the legs of the triangle in the diagram. If students write the first equation as $\frac{3-y}{1-x} = 2$, explain that $y-3$ and $x-1$ are equivalent to $-1(3-y)$ and $-1(1-x)$, respectively.

Student Task Statement

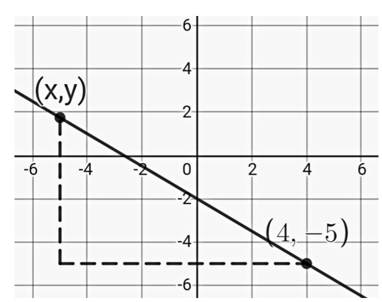
1. Find the slope of the line between the following sets of points:
   a. $(1, 6)$ and $(3, 8)$
   b. $(11, 6)$ and $(13, 9)$
   c. $(-4, 5)$ and $(-6, -1)$
   d. $(3, 7)$ and $(10, 0)$

2. The image shows a line.



   a. Find the slope between the two points $(1, 3)$ and $(3, 7)$ using the formula for slope.
   b. Using the same formula as above, write an equation that says the slope between the points $(1, 3)$ and (x, y) is 2.
   c. Look at this equation: $y - 3 = 2(x - 1)$
   How does it relate to the equation you wrote?
   d. How can you quickly tell from looking at this equation that $(1, 3)$ must be on the line?

3. The image shows a line.



   a. Find the slope between the two points $(4, -5)$ and $(0, -2)$ using the formula for slope.
   b. Using the same formula as above, write an equation that says the slope between the points $(4, -5)$ and (x, y) is $-\frac{3}{4}$.
   c. Look at this equation: $y + 5 = -\frac{3}{4}(x - 4)$. How does it relate to the equation you wrote?
   d. How can you quickly tell from looking at this equation that $(4, -5)$ must be on the line?

4. Here is an equation for another line: $y-7=\frac{1}{2}(x-5)$
   a. What point do you know this line passes through?
   b. What is the slope of this line?
5. Next, let's write a general equation that we can use for any line. Suppose we know a line passes through a particular point (h,k).
   a. Write an equation that says the slope between point (x,y) and (h,k) is m.
   b. Look at this equation: $y-k=m(x-h)$. How does it relate to the equation you wrote?

Step 2

The goal of the discussion is to ensure students understand the point-slope form of the equation of a line. Here are some questions for discussion:

- "What set of points does the equation $y-3=2(x-1)$ represent?" (It is the set of points that has a slope of 2 with the point $(1,3)$. That is, it's the line with slope 2 that goes through $(1,3)$.)
- "The equation $y-k=m(x-h)$ is called the point-slope form for the equation of a line. What do $(x,y),(h,k)$, and m represent?" (The ordered pair (x,y) represents any point on the line. The point (h,k) represents a point that we know is on the line. The letter m represents the slope of the line.)
- "Why do we subtract the k from the y and the h from the x?" (This gives the lengths of the legs of the slope triangle.)
- Remind students that that there are multiple ways to write the equation of a line, including slope-intercept form, $y=mx+b$, and standard form, $Ax+By=C$. For the rest of this unit, encourage students to use the equation form that is most strategic based on the information given.

RESPONSIVE STRATEGY

Use this routine to help students produce statements about the terms in the point-slope form equation of a line. Provide sentence frames for students to use when they interpret (x , y), (h , k), and m, such as: "____ represents ____." and "It looks like ____ represents...."

Discussion Supports (MLR8)

RESPONSIVE STRATEGY

Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of: slope, y-intercept, and point-slope form.

Supports accessibility for: Conceptual processing; Language

DO THE MATH

PLANNING NOTES

Activity 2: Using Point-Slope Form (*10 minutes*)

Instructional Routine: Compare and Connect (MLR7)	
Addressing: NC.M1.A-SSE.1a	**Building Towards:** NC.M1.G-GPE.5

This task allows students to practice writing and reading equations in point-slope form. Monitor for a variety of answers for the last part of the first question to highlight during step 3.

Students engage in the *Compare and Connect* routine to prepare for the whole-class discussion about the many equivalent ways to write an equation of a line.

COMPARE AND CONNECT	***What Is This Routine?*** The teacher facilitates a discussion about two or more pieces of student work that include distinct mathematical representations or approaches to a problem, calling attention to the correspondences among quantities, relationships, and features of the representations. Teachers should demonstrate thinking out loud (e.g., exploring why one might do or say it this way, questioning an idea, wondering how an idea compares or connects to other ideas or language), and students should be prompted to reflect and respond. ***Why This Routine?*** Use *Compare and Connect* (MLR7) to foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, and language. This routine supports meta-cognitive and meta-linguistic awareness, and also supports mathematical conversation.

Step 1

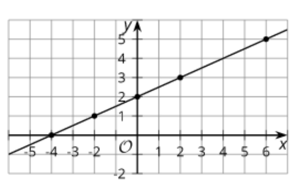
- Ask students to arrange themselves in pairs or use visibly random grouping to collaborate on question 1.
- After students find the equations of the graph of the lines, invite them to create a visual display of their work.

Monitoring Tip: Monitor for a variety of answers for the last part of the first question to highlight during Step 3.

Advancing Student Thinking: Students may incorrectly write $y-8=\frac{4}{5}(x-2)$ for the first equation (1a), not realizing they should use $x-(-2)$, or $x+2$. A picture may be helpful for these students: If x is some number to the right of the x-axis, then to find the horizontal side of the slope triangle, you should add 2.

Student Task Statement

1. Write an equation that describes each line.
   a. the line passing through point $(-2,8)$ with slope $\frac{4}{5}$
   b. the line passing through point $(0,7)$ with slope $-\frac{7}{3}$
   c. the line passing through point $(\frac{1}{2},0)$ with slope -1
   d. the line in the image:



Step 2

- Choose and display two-three different equations for part d, with the names of the creators to give credit. If a student uses slope-intercept form, make sure to include it for comparison. Create your own equation(s) in standard and/or slope-intercept form if needed, to add variety.
- Give students quiet think time to consider what is the same and what is different about their equation compared to the ones displayed. For example, how is the slope evident in each equation?
- Ask students to return to their partner to discuss what they noticed.

RESPONSIVE STRATEGIES
Leverage choice around perceived challenge. Invite students to select two parts to complete for each question. Chunking this task into more manageable parts may also help students who benefit from additional processing time.

Supports accessibility for: Organization; Attention; Social-emotional skills

Monitoring Tip: Listen for and amplify the language students use to compare and contrast the equations of the line.

- Prompt students to continue working together on question 2.

Advancing Student Thinking: If students struggle with identifying the point that the line passes through in 2b and 2c, suggest that they look back to question 1. For lines with points that included the number 0, how can those be rewritten so that the 0 doesn't appear? Do any of those forms look similar to the equations in the second question?

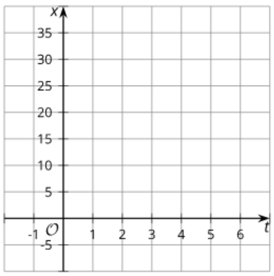
Student Task Statement

2. Using the structure of the equation, what point do you know each line passes through? What's the line's slope?
   a. $y-5=\frac{3}{2}(x+4)$
   b. $y+2=5x$
   c. $y=-2(x-\frac{5}{8})$

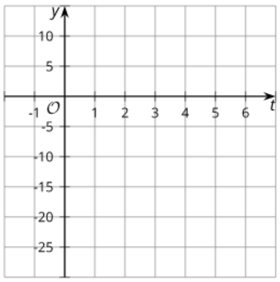
Are You Ready For More?

Another way to describe a line, or other graphs, is to think about the coordinates as changing over time. This is especially helpful if we're thinking about tracing an object's movement. This example describes the x- and y-coordinates separately, each in terms of time, t.

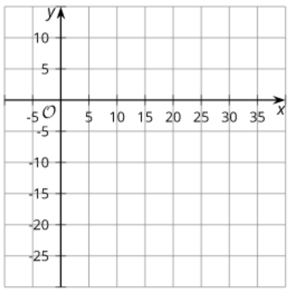
Grid A



Grid B



Grid C



1. On grid A, create a graph of $x=2+5t$ for $-2\le t\le 7$ with x on the vertical axis and t on the horizontal axis.
2. On grid B, create a graph of $y=3-4t$ for $-2\le t\le 7$ with y on the vertical axis and t on the horizontal axis.
3. On grid C, create a graph of the set of points $(2+5t, 3-4t)$ for $-2\le t\le 7$ on the xy-plane.

Step 3

- Facilitate a whole-class discussion on the last part of the first question by inviting previously identified students to share their answers. Record and display these answers for all to see, and then invite the class to identify what is the same and what is different about the equations. The goal of the discussion is to demonstrate that there are many equivalent ways to write an equation for any given line. Possible examples include:
  - $y-3=\frac{1}{2}(x-2)$
  - $y-5=\frac{1}{2}(x-6)$
  - $y=\frac{1}{2}x+2$
  - $y-0=\frac{1}{2}(x-(-4))$
- If time allows, graph each answer using Desmos. Point out that these are all different ways to describe the same line. Any point on the line can be substituted for (h,k), and the equation can be put into slope-intercept form by rearranging. Challenge students to choose two answers and rewrite the equations to show they are equivalent.

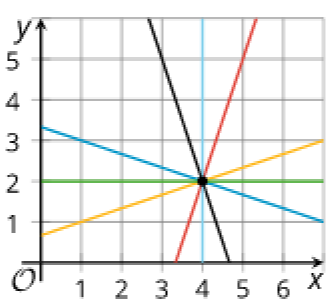
DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

In this lesson, students discover another form for the equation of a line that follows from their definition of slope. They also recognize equivalent equations for the same line.

Choose whether students will first have an opportunity to reflect in their workbooks or talk through these questions with a partner prior to the full class discussion.

Display this image for all to see:



Ask students:

- “What do you notice?” (All the lines intersect at $(4,2)$. One line is horizontal and one is vertical. The other 4 lines are either slanted upward or slanted downward.)
- “Write the equations of at least 3 different lines shown.”
  - $y=2$
  - $x=4$
  - $y-2=\frac{1}{3}(x-4)$

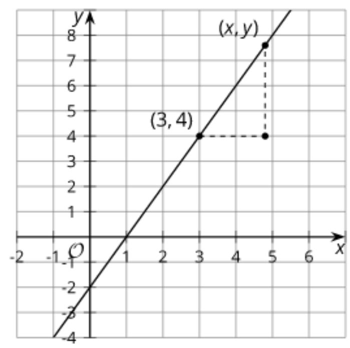
PLANNING NOTES

- $y-2=-\frac{1}{3}(x-4)$
- $y-2=-3(x-4)$
- $y-2=3(x-4)$

Invite students to share the equations they wrote. Record and display their responses for all to see. Use graphing technology to show that the equations they wrote match the image.

Ask which form of the equation of a line students prefer. (Any preference students state is valid. Sample responses: Slope-intercept is best, because it's easy to graph a line in this form. Point-slope is best, because you can use *any* point in it, not just the y-intercept.)

Student Lesson Summary and Glossary



The line in the image can be defined as the set of points that make a slope of 2 with the point $(3,4)$. The equation $\frac{y-4}{x-3}=2$ says the slope between points (x,y) and $(3,4)$ is 2. This equation can be rearranged to look like $y-4=2(x-3)$.

The equation is now in **point-slope form**, or $y-k=m(x-h)$, where:

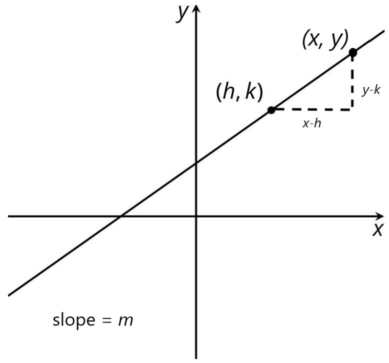
- (x,y) is any point on the line
- (h,k) is a particular point on the line that we choose to substitute into the equation
- m is the slope of the line

Other ways to write the equation of a line include slope-intercept form, $y=mx+b$, and standard form, $Ax+By=C$.

To write the equation of a line passing through $(3,1)$ and $(0,5)$, start by finding the slope of the line. The slope is $-\frac{4}{3}$ because $\frac{5-1}{0-3}=-\frac{4}{3}$. Substitute this value for m to get $y-k=-\frac{4}{3}(x-h)$. Now we can choose any known point on the line to substitute for (h,k). If we choose $(3,1)$, we can write the equation of the line as $y-1=-\frac{4}{3}(x-3)$.

We could also use $(0,5)$ as the point, giving $y-5=-\frac{4}{3}(x-0)$. We can rearrange the equation to see how point-slope and slope-intercept forms relate, getting $y=-\frac{4}{3}x+5$. Notice $(0,5)$ is the y-intercept of the line. The graphs of all 3 of these equations look the same.

Point-slope form: The form of a linear equation written as $y-k=m(x-h)$, where m is the slope of the line and (h,k) is a point on the line. Point-slope form can also be written as $y=k+m(x-h)$.



Cool-down: Same Slope, Different Point (*5 minutes*)

Addressing: NC.M1.A-SSE.1a	**Building Towards:** NC.M1.G-GPE.5
Cool-down Guidance: More Chances Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.	

Cool-down

Consider the line represented by $y + 4 = \frac{2}{5}(x - 9)$. Write an equation representing a different line with the same slope that passes through the point $(3, 6)$.

Student Reflection:

Imagine your classmate was absent today. How would you explain to them what you learned today?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

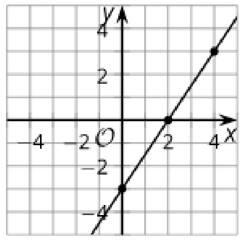
TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

With which math ideas from today's lesson did students grapple most? Did this surprise you, or was this what you expected?

Practice Problems

1. Select **all** the equations that represent the graph shown.



   a. $3x - 2y = 6$
   b. $y = \frac{3}{2}x + 3$
   c. $y = \frac{3}{2}x - 3$
   d. $y - 3 = \frac{3}{2}(x - 4)$
   e. $y - 6 = \frac{3}{2}(x - 2)$

2. Write the equation $y + 2 = 3(x + 1)$ in slope-intercept form.

3. A line with slope $\frac{3}{2}$ passes through the point $(1, 3)$.
   a. Explain why $(3, 6)$ is on this line.
   b. Explain why $(0, 0)$ is not on this line.
   c. Is the point $(13, 22)$ on this line? Explain why or why not.

4. Write an equation of the line that passes through $(1, 3)$ and has a slope of $\frac{5}{4}$.

5. Write two equivalent equations for a line with x-intercept $(3, 0)$ and y-intercept $(0, 2)$.

6. Clare has been working to save money and wants to have an equation to model the amount of money in her bank account. She has been depositing $175 a month consistently. She doesn't remember how much money she deposited initially; however, on her last statement she saw that her account has been open for 10 months and currently has $2475 in it. Write an equation for the amount of money in Clare's bank account after x months. Which equation form did you choose?[1]

7. Solve for the indicated variable in each part.[2]
   a. $x : ax = 7$
   b. $p : 8 + p = w$
   c. $y : \frac{1}{2}y = k$
   d. $x : y = mx + b$

   (From Unit 2)

8. Elena's mother's painting service charges $10 per job and $0.20 per square foot. If she earned $50 for painting one job, how many square feet did she paint at the job? Write an equation and solve.[3]

   (From Unit 2)

9. Han took a math test with 20 questions, and each question is worth an equal number of points. The test is worth 100 points total.[4]
   a. Write an equation that can be used to calculate Han's score based on the number of questions he got right on the test.
   b. If a score of 70 points earns a grade of a C, how many questions would Han need to get right to get at least a C on the test?
   c. If a score of 80 points earns a grade of B, how many questions would Han need to get at least a B on the test?
   d. Suppose Han got a score of 60% and then chose to retake the test. On the retake, Han got all the questions right that he got right the first time, and he also got half the questions right that he got wrong the first time. What percent of the questions did Han get right, in total, on the retake?

   (From Unit 2)

[1] Adapted from Math 1, Module 2, Lesson 10 Mathematics Vision project http://www.mathematicsvisionproject.org, licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0)
[2] Adapted from Math 1, Module 4 Mathematics Vision project http://www.mathematicsvisionproject.org (see above).
[3] Adapted from Math 1, Module 4 Mathematics Vision project http://www.mathematicsvisionproject.org (see above).
[4] Adapted from Math 1, Module 4 Mathematics Vision project http://www.mathematicsvisionproject.org (see above).

Lesson 5: Equations of Parallel Lines

PREPARATION

Lesson Goals	Learning Targets
• Establish the slope criterion for parallel lines. • Write the equation of a line parallel to a given line that passes through a given point.	• I can determine if two lines are parallel. • I can write the equation of a line parallel to a given line that passes through a given point.

Lesson Narrative

In previous lessons, students have analyzed how the numbers in a linear equation are related to the rate of change in the relationship between two variables, and how these numbers are connected to the graph of the equation. In particular, they have identified the slope of the line from the equation. In some cases, such as when the equation is presented in the form $ax + by = c$, students may have rewritten the equation into a different form to identify the slope.

In this lesson, students will begin by comparing the slopes of parallel lines and recognizing that parallel lines have the same slope. Next, given the graph of a line, students graph another line that is parallel. In this process, students develop an understanding that if two lines have at least one point not in common and are increasing at the same rate, then the lines will never intersect. They make a conjecture about the slopes of parallel lines and construct viable arguments to support the conjecture (MP3). Once students are convinced that parallel lines have equal slopes, they apply this knowledge to write equations of lines that are parallel to a given line and pass through a given point (MP7).

Share some ways you see this lesson connecting to previous lessons in this unit. What connections will you want to make explicit?

Focus and Coherence

Building On	Addressing
NC.M1.A-CED.2: Create and graph equations in two variables that represent linear, exponential, and quadratic relationships and use them to solve problems. **NC.4.G.1:** Draw and identify points, lines, line segments, rays, angles, and perpendicular and parallel lines.	**NC.M1.G-GPE.5:** Use coordinates to prove the slope criteria for parallel and perpendicular lines and use them to solve problems. • Determine if two lines are parallel, perpendicular, or neither. • Find the equation of a line parallel or perpendicular to a given line that passes through a given point. **NC.M1.A-SSE.1a:** Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.

Agenda, Materials, and Preparation

- **Bridge** *(Optional, 5 minutes)*
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
    - M1.U3.L5 Cool-down (print 1 copy per student)

LESSON

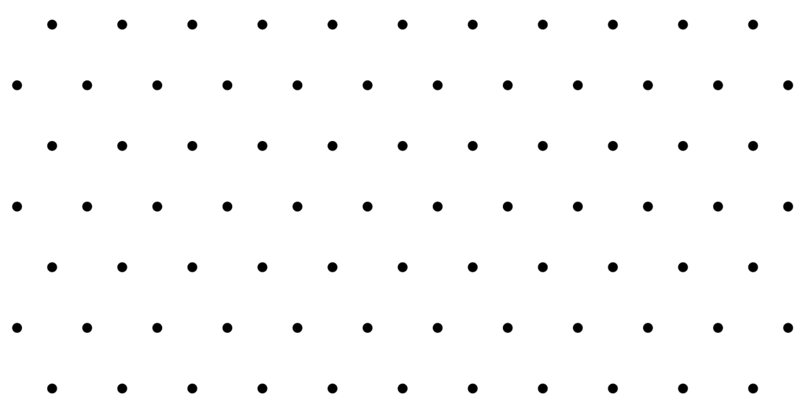
Bridge (*Optional, 5 minutes*)

Building On: NC.4.G.1

The purpose of this bridge is for students to draw intersecting and parallel lines, and to explain how they know a pair of parallel lines would never intersect. Students are expected to make a case that goes beyond appearance (such as "it looks like they would never cross") and notice that the parallel lines maintain the same distance apart (MP3). This task is aligned to question 2 in Check Your Readiness.

Student Task Statement

Here is a field of dots. Each dot represents a point.[1]



1. Draw a line through at least two points. Label it line h.
2. Draw another line that goes through at least two points and intersects your first line. Label it line g.
3. Can you draw a new line that you think would never intersect:
    a. line h?
    b. line g?

    If so, draw the line. Be prepared to explain or show how you know the lines would never cross. If not, explain or show why it can't be done.

[1] Adapted from IM K–5, Unit 7, Lesson 3 https://curriculum.illustrativemathematics.org/K5/index.html,

DO THE MATH	PLANNING NOTES

Warm-up: Three Lines (*5 minutes*)

Instructional Routine: Notice and Wonder	
Building On: NC.M1.A-CED.2	**Building Towards:** NC.M1.G-GPE.5

The purpose of the warm up is to elicit the idea that parallel lines have the same slope. While students may notice and wonder many things about the graphs and equations, slope of the lines and the appearance of the lines being parallel are important discussion points.

NOTICE AND WONDER	***What Is This Routine?*** Students are shown some media or a mathematical representation. The prompt to students is "What do you notice? What do you wonder?" Students are given a few minutes to think of things they notice and things they wonder, and share them with a partner. Then, the teacher asks several students to share things they noticed and things they wondered; these are recorded by the teacher for all to see. Sometimes the teacher steers the conversation to wondering about something mathematical that the class is about to focus on. ***Why This Routine?*** The purpose of the *Notice and Wonder* routine is to make a mathematical task accessible to all students with these two approachable questions. By thinking about them and responding, students gain entry into the context and might get their curiosity piqued. Taking steps to become familiar with a context and the mathematics that might be involved is making sense of problems (MP1).

Step 1

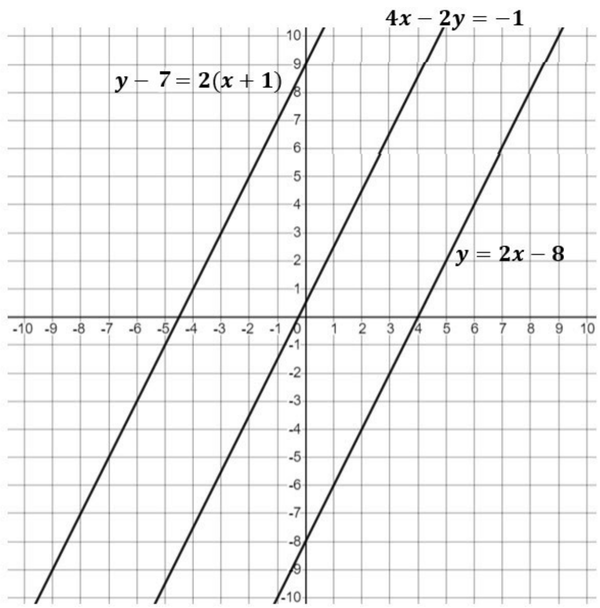
- Display the graph of the three lines and their equations.
- Ask students to think of at least one thing they notice and at least one thing they wonder.
- Give students 1 minute of quiet think time.

Step 2

- Give students 1 minute to discuss the things they notice and wonder with their partner.

Student Task Statement

The lines of three linear equations are graphed below. What do you notice? What do you wonder?



Step 3

- Facilitate a whole-class discussion by asking students to share the things they noticed and wondered. The goal of the discussion is to help students recognize that all three lines have the same slope. This can be verified by drawing slope triangles, calculating slope using points on the line, or by identifying the slope from the equation. The three lines also appear to be parallel. For the second line, students may rightly be skeptical of graphical methods, since we cannot see exactly which points the line passes through.
- Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image.
- After all responses have been recorded without commentary or editing, ask students, "Is there anything on this list that you are wondering about now?" Encourage students to respectfully disagree, ask for clarification, or point out contradicting information.

DO THE MATH	PLANNING NOTES

Activity 1: Graphing Parallel Lines (*15 minutes*)

Instructional Routine: Stronger and Clearer Each Time (MLR1) - Responsive Strategy	
Building On: NC.M1.A-CED.2	**Addressing:** NC.M1.G-GPE.5

In this activity, students will graph a line that is parallel to a given line. They will compare their line to their partner's and decide if the lines they graphed are also parallel to each other. After a whole class discussion on the slope criteria for parallel lines or the use of the responsive strategy *Stronger and Clearer Each Time* routine, students will continue to work with a partner to determine if pairs of lines are parallel.

STRONGER AND CLEARER EACH TIME	***What Is This Routine?*** Students write a first draft response to a prompt, then engage in successive pair-shares to have multiple opportunities to refine and clarify their response through conversation, and then finally revise their original response. Throughout this process, students should be encouraged to press each other for clarity and details. ***Why This Routine?*** *Stronger and Clearer Each Time* (MLR1) provides a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output. The routine provides a purpose for student conversation and fortifies student output.

Step 1

- Ask students to arrange themselves into pairs or use visibly random grouping.
- Give students 2 minutes of quiet think time to graph a line. Afterwards, ask students to compare their lines to their partner's and decide if both are correct, even if they are different. Direct them to make any adjustments needed.

RESPONSIVE STRATEGY

Provide a manipulative such as a toothpick, dry spaghetti noodle, or clear ruler to allow students to try different lines before deciding on a parallel line.

Supports accessibility for: Visual-Spatial Processing

Monitoring Tip: Monitor for these likely strategies for graphing the line parallel to the given line.

- Approximating visually to graph what appears to be a parallel line.
- Selecting points on the line and translating them vertically or horizontally.
- Selecting a point not on the line and using slope to locate another point on the line.

Identify students who use each strategy and let them know that they may be asked to share later. Include at least one student who does not typically volunteer.

Step 2

- Pause for a whole-class discussion.
- Select previously identified students to share their strategies, in the order listed in the Monitoring Tip. If one of the strategies is not mentioned, bring it up. Ask students when they share their strategies why it graphs a parallel line.
  - By translating the points, the new line will always be the same distance from the given line, meaning they will never intersect. Lines that don't intersect are parallel.
  - With both lines changing at the same rate one will never be steeper or less steep than the other, so they will never intersect.

RESPONSIVE STRATEGY

Instead of a whole-class discussion, use the Stronger and Clearer Each Time routine. Give students 1 minute to make some initial notes about how they know their line is parallel to the given line, then pair them up twice -- each time with a different partner -- to engage in two rounds of partner sharing and feedback, and finally give them 1 minute to revise their notes into a stronger and clearer justification. Their improved notes can be considered a 'second draft'; this routine will support students to generate more robust justifications for question 2.

Stronger and Clearer Each Time (MLR1)

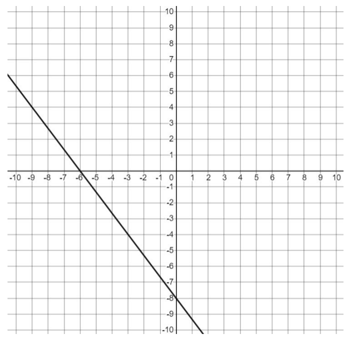
Step 3

- Ask students to continue to work in pairs and complete the remainder of the activity.

***Advancing Student Thinking*:** In the first question, students may think that there is a specific line they need to graph. Tell them there are many possible answers. If needed, suggest they plot a point anywhere not on the original line and graph a line through that point that is parallel to the given line.

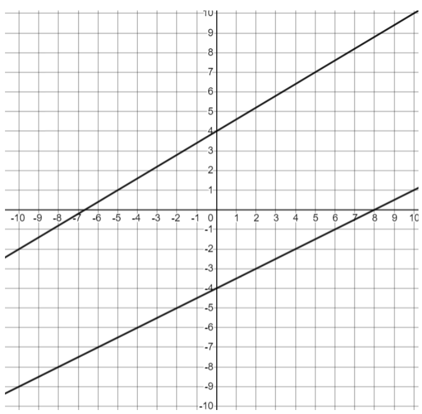
Student Task Statement

1. Given this graph of a linear equation, graph a line that is parallel. Be prepared to explain your process.



2. For each of the following pairs of lines, determine if the lines are parallel or not. Provide justification for your response.

a.



b. $y = 3x - 7$

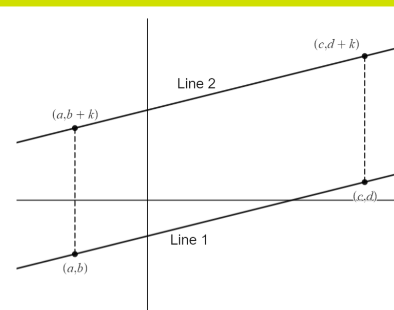
$6x - 2y =- 8$

Are You Ready For More?

The following statements relate to the graph:

- Points (a,b) and $(c.d)$ are on Line 1.
- Each point was translated k units up.
- Points $(a,b+k)$ and $(c,d+k)$ are on Line 2.

Prove that the slopes of the lines are the same.



Step 4

- Ask students to share their responses and justifications to whether or not the pairs of lines are parallel. Highlight responses that compared the slopes of the lines.

DO THE MATH	PLANNING NOTES

Activity 2: Writing Equations of Parallel Lines (*10 minutes*)

Building On: NC.M1.A-CED.2	**Addressing:** NC.M1.G-GPE.5; NC.M1.A-SSE.1a

In this activity, students practice writing the equation of a line parallel to a given line and passing through a given point.

Making graphing technology available but not suggesting it gives students an opportunity to choose appropriate tools strategically (MP5).

Step 1

- Keep students in pairs.
- Tell students they will now write equations of lines that are parallel to a given line and passing through a given point.
- Provide students 2–3 minutes of quiet think time and then time to share their thinking with their partner.

Monitoring Tip: As students work, notice the strategies they use to identify the slope of the given line and how they write the equation of the parallel line. Also note which students rewrite their equation into a different form.

Advancing Student Thinking: If students are not sure how to write the equation of the parallel line, remind them of the point-slope form of a linear equation from the previous lesson. Ask what information they need to use point-slope form and how they could identify the information from what has been given.

Student Task Statement

1. Write the equation of a line that is parallel to $y = 5x + 4$ and passes through the point $(3, -2)$.
2. Write the equation of a line that is parallel to $2x + 3y = 4$ and passes through the point $(3, 1)$.

Step 2

- Select students who use different strategies to share their responses and reasoning. Start with the most common strategy and then share alternative approaches. Since students will have additional opportunities to strengthen their understanding in future lessons, focus on selecting strategies that are representative of student thinking even if the strategies are not entirely correct and include misconceptions. This will contribute to a classroom culture in which making mistakes is part of the learning process, rather than reinforce negative connotations around making errors.
- If possible, display their work for all to see. After each student presents, ask if others solved it the same way and model checking for accuracy using a digital graphing tool such as Desmos. If the two graphed lines are not parallel, encourage students to think about where the strategy may have been flawed.

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

In this lesson, students establish the slope criteria for parallel lines. They use the criteria to determine if lines are parallel and to write the equation of a line that is parallel. Facilitate a discussion using the following questions. As students respond, make connections to examples from the lesson.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion. • "How would you explain to another student why parallel lines have the same slope?" • "Which form(s) of the equation makes it easiest to identify the slope?" (slope intercept or point slope) • "How do you identify the slope when it is not in one of these forms? For example, when given 2x + 5y = 15." (rewrite the equation by solving for y) • "What form of the equation do you prefer when writing the equation of a line?"	**PLANNING NOTES**

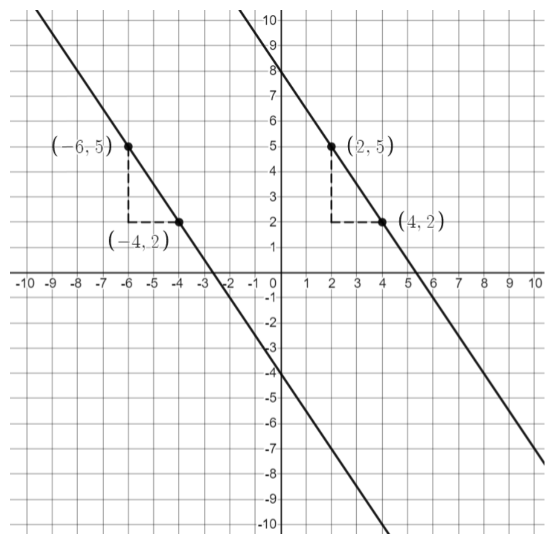
Student Lesson Summary and Glossary

Two lines on a coordinate plane are parallel if and only if they have the same slope. This means:

- If two lines are parallel, then their slopes are the same.
- If the slopes of two lines are the same, then the lines are parallel.

To determine if two lines are parallel, compare the slopes of the lines.

- In the graph, slope triangles are used to identify the slope. The two lines graphed have a slope of $\frac{-3}{2}$, so they are parallel.
- Given two linear equations, $y = 2x + 9$ and $8x - 4y = 12$:
  - The slope of the line defined by the first equation is 2.
  - To find the slope of the second equation, first rewrite it as as $y = 2x - 3$.
  - The slope of the line defined by the second equation is 2.
  - The slopes of each line are the same so the lines are parallel.



To write the equation of a line that is parallel to the line $y = \frac{4}{5}x - 7$ and passes through a point $(15, 2)$, start by identifying the slope. The slope of the line is $\frac{4}{5}$. Using point-slope form, the equation of the parallel line is $y - 2 = \frac{4}{5}(x - 15)$ which can be rewritten as $y = \frac{4}{5}x - 10$.

Cool-down: Parallel and Passing Through (*5 minutes*)

Addressing: NC.M1.G-GPE.5

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding. Students will work to apply these ideas in Lessons 8, 9, and 10. Look for ways to amplify connections.

Cool-down

Write the equation of a line parallel to $x + 4y = 3$ and passing through the point $(8, -4)$. Explain or show your reasoning.

Student Reflection:

I felt confident/joyful during this lesson when____

and I felt less confident/joyful during this lesson when ____________________________________.

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

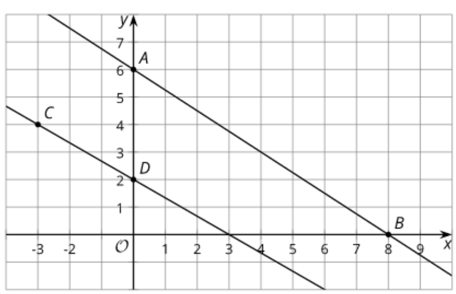
NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

With which math ideas from today's lesson did students grapple most? Did this surprise you or was this what you expected?

Practice Problems[2]



1. The image shows lines AB and CD.

   Are the two lines parallel? Explain or show your reasoning.

2. Select **all** equations that are parallel to the line $2x + 5y = 8$.
   a. $y = \frac{2}{5}x + 4$
   b. $y = -\frac{2}{5}x + 4$
   c. $y - 2 = \frac{2}{5}(x + 1)$
   d. $y - 2 = -\frac{2}{5}(x - 1)$
   e. $10x + 25y = 40$

3. Write an equation of a line that passes through $(-1, 2)$ and is parallel to a line with x-intercept $(3, 0)$ and y-intercept $(0, 1)$.

[2] Adapted from IM 9–12 Math https://curriculum.illustrativemathematics.org/HS/index.html,

4. Write an equation of the line with slope $\frac{2}{3}$ that goes through the point $(-2, 5)$.

   (From Unit 3, Lesson 4)

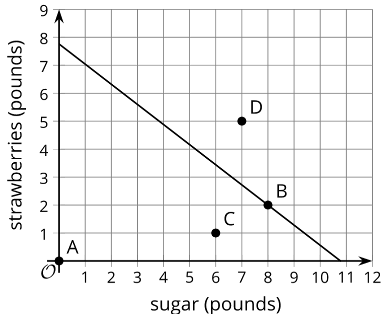
5. Priya and Han each wrote an equation of a line with slope $\frac{1}{3}$ that passes through the point $(1, 2)$. Priya's equation is $y - 2 = \frac{1}{3}(x - 1)$ and Han's equation is $3y - x = 5$. Do you agree with either of them? Explain or show your reasoning.

   (From Unit 3, Lesson 4)

6. Jada brought some sugar and strawberries to make strawberry jam. Sugar costs $1.80 per pound, and strawberries cost $2.50 per pound. Jada spent a total of $19.40.

   Which point on the coordinate plane could represent the pounds of sugar and strawberries that Jada used to make jam?

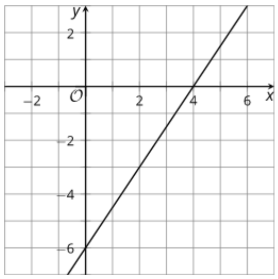
   (From Unit 3, Lesson 1)



7. Here is a graph of the equation 3x-2y=12. Select **all** coordinate pairs that represent a solution to the equation.

   a. (2,-3)
   b. (4,0)
   c. (5,-1)
   d. (0,-6)
   e. (2,3)

   (From Unit 3, Lesson 1)



8. The graph shows the relationship between temperature in degrees Celsius and temperature in degrees Fahrenheit.

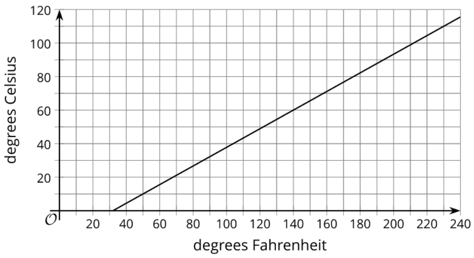
   a. Mark the point on the graph that shows the temperature in Celsius when it is 60 degrees Fahrenheit.

   b. Mark the point on the graph that shows the temperature in Fahrenheit when it is 60 degrees Celsius.

   c. Water boils at 100 degrees Celsius. Use the graph to approximate the boiling temperature in Fahrenheit, or to confirm it, if you know what it is.

   d. The equation that converts Fahrenheit to Celsius is $C = \frac{5}{9}(F - 32)$. Use it to calculate the temperature in Celsius when it is 60 degrees Fahrenheit. (This answer will be more exact than the point you found in the first part.)

   (From Unit 3, Lesson 1)



9. A cell phone company offers a plan that costs $35.99 and includes unlimited texting. Another company offers a plan that costs $19.99 and charges $0.25 per text. For what number of texts does the second company's plan cost more than the first company's plan?[3]

   a. Write and solve an inequality that models this situation. Be sure to define your variables.

   b. Describe in words the quantities that would work in this situation.

   (From Unit 2)

[3] Adapted from Math 1, Module 4, Lesson 4.5 Mathematics Vision project http://www.mathematicsvisionproject.org, licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0)

Lesson 6: Equations of Perpendicular Lines

PREPARATION

Lesson Goals	Learning Targets
• Establish the slope criterion for perpendicular lines. • Write the equation of a line perpendicular to a given line that passes through a given point.	• I can determine if two lines are perpendicular. • I can write the equation of a line perpendicular to a given line that passes through a given point.

Lesson Narrative

In the previous lesson, students established the slope criterion for parallel lines. In this lesson, students will establish the slope criterion for perpendicular lines. They will continue to apply their understanding and procedural skill of writing the equations of lines.

The lesson begins with students exploring the structure of multiplying reciprocals. Students preview the structure (MP8) and vocabulary that will be used in the rest of the lesson. The next activity presents right triangles graphed on the coordinate plane. Students analyze the slopes of the sides that form the right angle and make a conjecture about the slopes of the perpendicular lines (MP3). In the second activity, students are given a line on the coordinate plane and write the equation of a perpendicular line that passes through a given point.They will also analyze a common error when writing equations of perpendicular lines.

The use of right triangles in this lesson serves as a preview to learning later in this course when students use coordinates to verify types of triangles and quadrilaterals.

How is the approach of this lesson similar and different from other ways you have taught these concepts or procedures?

Focus and Coherence

Building On	Addressing
NC.4.G.1: Draw and identify points, lines, line segments, rays, angles, and perpendicular and parallel lines. **NC.M1.A-CED.2:** Create and graph equations in two variables that represent linear, exponential, and quadratic relationships and use them to solve problems.	**NC.M1.G-GPE.5:** Use coordinates to prove the slope criteria for parallel and perpendicular lines and use them to solve problems. • Determine if two lines are parallel, perpendicular, or neither. • Find the equation of a line parallel or perpendicular to a given line that passes through a given point. **NC.M1.A-SSE.1a:** Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.

Agenda, Materials, and Preparation

- **Bridge** *(Optional, 5 minutes)*
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
    - M1.U3.L6 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.4.G.1

The purpose of this bridge is to revisit the definition of perpendicular lines and identify perpendicular segments in two-dimensional figures. Students may use characteristics of polygons such as rectangles and right triangles to help identify the perpendicular line segments. This task is aligned to question 2 in Check Your Readiness.

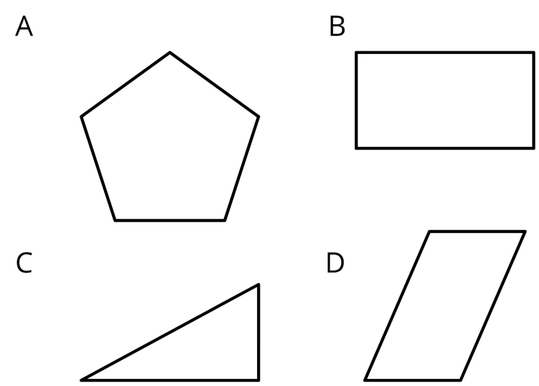
RESPONSIVE STRATEGIES

Create an anchor chart to provide visual example for perpendicular lines. Ask students to provide examples of polygons with perpendicular lines.

Student Task Statement

Which shapes have sides that are perpendicular to one another?[1]

Mark the perpendicular sides. How would you explain the word "perpendicular" to a sixth grader?



DO THE MATH

PLANNING NOTES

[1] Adapted from IM K–5, Grade 4, Unit 7, Lesson 10 https://curriculum.illustrativemathematics.org/K5/index.html, copyright 2019 by Illustrative Mathematics. Licensed under the Creative Commons Attribution 4.0 license https://creativecommons.org/licenses/by/4.0/.

Warm-up: Special Products (*5 minutes*)

Instructional Routines: Which One Doesn't Belong?; Discussion Supports (MLR8) - Responsive Strategy	
Building Towards: NC.M1.G-GPE.5	

This warm-up prompts students to carefully analyze and compare products of reciprocals using the *Which One Doesn't Belong?* routine. In making comparisons, students have a reason to use language precisely (MP6). They may also recognize and make use of the structure of the expressions (MP8).

The work here prepares students to recognize reciprocals, building towards identifying opposite reciprocals for slopes of perpendicular lines.

Step 1

- Ask students to arrange themselves in small groups or use visibly random grouping.
- Display the expressions for all to see.
- Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, ask each student to share their reasoning as to why a particular expression does not belong, and together find at least one reason each item doesn't belong.

Student Task Statement

Which one doesn't belong? Explain your reasoning.

a. $8 \times \frac{1}{8}$	b. $\frac{2}{3} \times \frac{3}{2}$
c. $-\frac{4}{7} \times -\frac{7}{4}$	d. $\frac{5}{6} \times -\frac{6}{5}$

Step 2

- Ask each group to share one reason why a particular item does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given are correct.
- During the discussion, ask students to explain the meaning of any terminology they use, such as "flipped" or "inverse." Ask them if there is a mathematical term for this concept (reciprocal).

RESPONSIVE STRATEGY

Provide sentence frames for students to leverage during the share-out, such as "This doesn't belong because...," "I agree because...," or "I disagree because...."

Discussion Supports (MLR8)

DO THE MATH	PLANNING NOTES

Activity 1: Make a Conjecture *(15 minutes)*

Instructional Routine: Collect and Display (MLR2)	
Addressing: NC.M1.G-GPE.5	

Students calculate the slopes of the line segments that form the right angle of the right triangles. They make a conjecture for the slope criteria for perpendicular lines.

Step 1

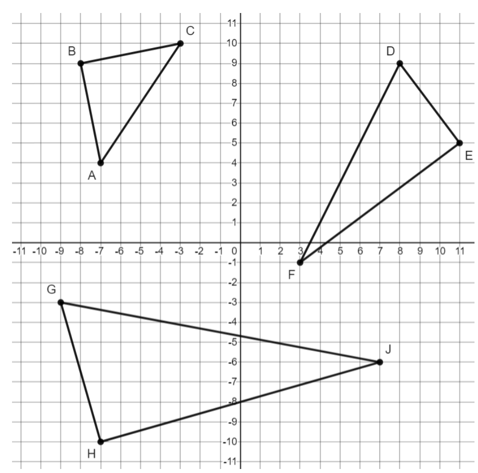
- Tell students that they will be exploring the slopes of perpendicular line segments.
- Ask students to arrange themselves into groups of three or use visibly random grouping. Each student in the group should complete the table for one of the right triangles. Students then take turns explaining how they calculated the slopes and the product before discussing the second question together.
    - Use the *Collect and Display* routine while circulating and listening as students work. Listen carefully for informal, everyday language and creative uses of language, and prepare to refer back to student words and phrases during the whole class discussion.

Monitoring Tip: As students work, notice whose conjecture is that the product of the slopes is -1, and for those who mention something about the structure of the two slopes, such as the fact that they have opposite signs or that they are "flipped."

***Advancing Student Thinking*:** Students may not know what it means to make a conjecture. Share that a conjecture is a mathematical statement that is suspected to be true based on evidence but has not been proven.

Student Task Statement

The image shows three right triangles where angles B, E, and H are right angles.



1. Complete the tables with the slopes of the line segments that form each right angle.

Triangle ABC		
Slope of side AB	**Slope of side BC**	**Product**

Triangle DEF		
Slope of side DE	**Slope of side EF**	**Product**

Triangle GHJ		
Slope of side GH	**Slope of side HJ**	**Product**

2. Use your slope calculations to make a conjecture about slopes of perpendicular lines.

Step 2

- Select previously identified students to share in this order:
  - First, invite a student whose conjecture involved the structures of the slopes to share their idea. Invite students to revisit any language collected and displayed and challenge them to use more precise language. For example, if they say the slopes are “flipped” or “upside down,” ask them if there is a mathematical term for this concept (reciprocal). Students may want to describe the slopes of perpendicular lines as negative reciprocals. Opposite reciprocals is clearer language—if the original slope is negative (such as $-\frac{4}{3}$), we can avoid the awkward $-(-\frac{4}{3})$ and instead jump directly to $\frac{4}{3}$.
  - Next, invite a student whose conjecture involved the product of the slopes being -1 to share. Ask students if this conjecture is different from the previous one. Explore this idea by presenting a slope of $\frac{a}{b}$ to students, and asking them to provide the opposite reciprocal ($-\frac{b}{a}$). Instruct students to find the product of these fractions (-1). The conjectures are equivalent, because the value that multiplies with $\frac{a}{b}$ to create -1 is $-\frac{b}{a}$.
- If it hasn’t come up, tell students there is at least one exception that doesn’t fit the conjecture: a pair of horizontal and vertical lines. Because a vertical line has no slope, the idea of an opposite reciprocal doesn’t make sense.
- Finally, present students with the following slopes of three different lines: $\frac{4}{7}$, 6, $\frac{-1}{3}$. Challenge students to identify the slope of a line perpendicular to each one.

DO THE MATH

PLANNING NOTES

Activity 2: Writing Equations of Perpendicular Lines (*10 minutes*)

Building On: NC.M1.A-CED.2	**Addressing:** NC.M1.G-GPE.5; NC.M1.A-SSE.1a

In this activity, students are given a line graphed on the coordinate plane. They write the equation of the line perpendicular to the given line and passing through the point P. Next, students identify an error in writing a perpendicular line and correct the error.

Making graphing technology available but not suggesting it gives students an opportunity to choose appropriate tools strategically (MP5).

Step 1

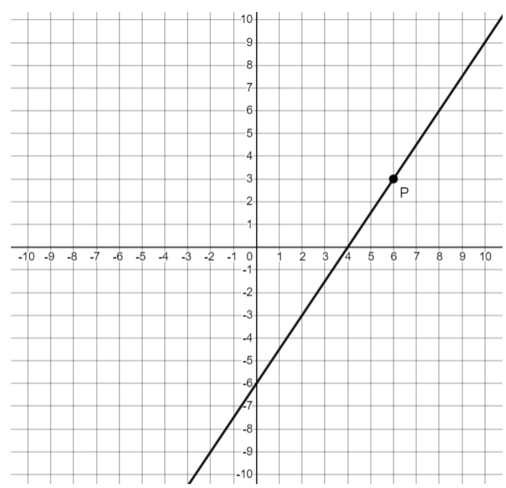
- Ask students to arrange themselves into pairs with someone that was not in their group for the last activity or use visibly random grouping.
- Provide students with 2–3 minutes of quiet think time to work on the task.
- Ask students to discuss their equations and responses to the task.
  - For question 1, students compare their equations and reconcile any differences.
  - For question 2, students take turns explaining. Partner 1 will explain to Partner 2 what mistake was made. Then Partner 2 will explain how to correctly write the equation to Partner 1.

Advancing Student Thinking: Students may struggle with applying both aspects of opposite reciprocals either by not making the slope opposite in sign or by not making it the reciprocal of the original slope. Encourage students to graph the lines using technology and visually check if the lines appear perpendicular.

Student Task Statement

1. Write the equation of the line perpendicular to the given line that passes through the point P.

2. Kiran tried to write an equation for the line perpendicular to $y = -3x + 2$ that passes through the point $(-4, 2)$. His answer was $y - 2 = 3(x + 4)$.
   a. The line is not perpendicular. Explain what Kiran's mistake was..
   b. What is the correct equation of the line?



Are You Ready For More?[2]

1. Line l is represented by the equation $y = \frac{2}{3}x + 3$. Write an equation of the line perpendicular to l, passing through $(-6, 4)$. Call this line p.
2. Write an equation of the line perpendicular to p, passing through $(3, -2)$. Call this line n.
3. What do you notice about lines l and n? Does this always happen? Show or explain your work.

[2] Adapted from IM 9–12 Math Geometry, Unit 6, Lesson 12 https://curriculum.illustrativemathematics.org/HS/index.html,

DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

In this lesson students establish the slope criteria for perpendicular lines. They use the criteria to determine if lines are perpendicular and to write the equation of a line that is perpendicular. Facilitate a discussion using the following questions. As students respond, make connections to examples from the lesson.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

- "What is the slope criterion for two lines to be perpendicular?" (The slopes of the lines must be opposite reciprocals or have a product of -1.)
- Provide equations of several lines such as $y = -4x + 2$, $2x + y = 9$ and $y = \frac{3}{5}x - 8$ and ask students to identify the slope of a perpendicular line.

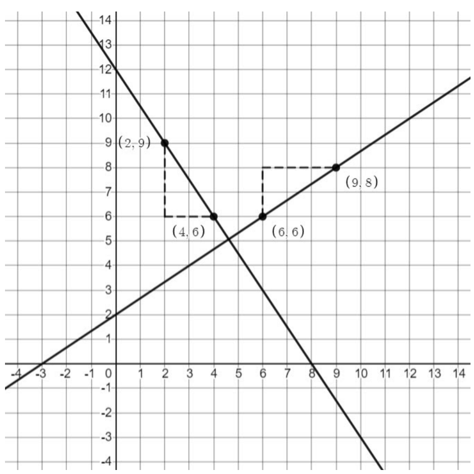
PLANNING NOTES

Student Lesson Summary and Glossary

Two lines on a coordinate plane are perpendicular if and only if their slopes are opposite reciprocals. This means:

- If two lines are perpendicular then their slopes are opposite reciprocals.
- If the slopes of two lines are opposite reciprocals then the lines are perpendicular.

Opposite reciprocals: A pair of numbers that can be expressed as $\frac{a}{b}$ and $-\frac{b}{a}$ where $a \neq 0$ and $b \neq 0$. Opposite reciprocals have a product of -1.



To determine if two lines are perpendicular, compare the slopes of the lines.

- In the graph, slope triangles are used to identify the slope. One line has a slope of $\frac{-3}{2}$ and the other has a slope of $\frac{2}{3}$. The slopes are opposite reciprocals so the lines are perpendicular.
- Given two linear equations, $y = 5x + 1$ and $2x + 10y = -20$:

- The slope of the line defined by the first equation is 5, which can be written as $\frac{5}{1}$.
- To find the slope of the second equation, first rewrite it as as $y=-\frac{1}{5}x-2$.
- The slope of the line defined by the second equation is $-\frac{1}{5}$.
- The slopes of $\frac{5}{1}$ and $-\frac{1}{5}$ are opposite reciprocals so the lines are perpendicular.

To write the equation of a line that is perpendicular to the line $y=\frac{3}{8}x+2$ and passes through the point $(-6,1)$, start by identifying the slope. The slope of the line is $\frac{3}{8}$. The slope of the perpendicular line is the opposite reciprocal $-\frac{8}{3}$. Using point-slope form, the equation of the perpendicular line is $y-1=-\frac{8}{3}(x+6)$, which can be rewritten as $y=-\frac{8}{3}x-15$.

Cool-down: Another Perpendicular Line (*5 minutes*)

Addressing: NC.M1.G-GPE.5

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding. Students will work to apply these ideas in Lessons 7, 8, and 9. Look for ways to amplify connections.

Cool-down

Write an equation of the line that passes through $(-4,1)$ and is perpendicular to the line $y=2x+3$.

Student Reflection:

When and/or where do you see parallel and perpendicular lines in the world around you?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

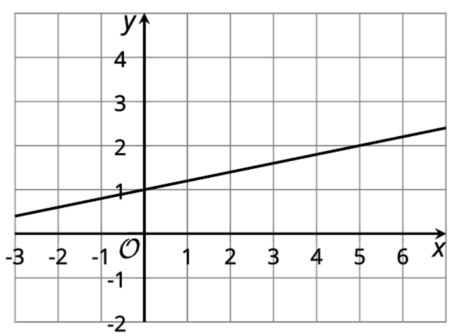
As students worked in their small groups today, whose ideas were heard, valued, and accepted? How can you adjust the group structure tomorrow to ensure each student's ideas are a part of the collective learning?

Practice Problems

1. Write an equation for a line that passes through the origin and is perpendicular to $y = 5x - 2$.
2. Match each line with a perpendicular line.

a. $y = 5x + 2$	1. the line through $(2, 12)$ and $(17, 9)$
b. $y - 2.25 = -2(x - 2)$	2. $y = -\frac{1}{2}x + 5$
c. the line through $(-1, 5)$ and $(1, 9)$	3. $2x - 4y = 10$

3. For each equation, is the graph of the equation parallel to the line shown, perpendicular to the line shown, or neither? Put a checkmark in the appropriate column.[3]



Equation	Parallel	Perpendicular	Neither
a. $y = 0.2x$			
b. $y =- 2x + 1$			
c. $y = 5x–3$			
d. $(y–3) =- 5(x–4)$			
e. $(y–1) = 2(x–3)$			
f. $5x + y = 3$			

[3] Adapted from IM 9–12 Math Geometry, Unit 6, Lesson 12 https://curriculum.illustrativemathematics.org/HS/index.html,

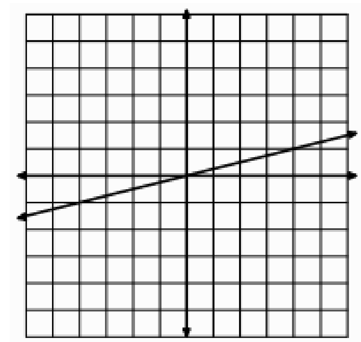
4. Line p has an x-intercept at $(3,0)$ and y-intercept at $(0,-4)$.

   a. Write the equation of the line p.

   b. Write the equation of the line parallel to line p that passes through $(2,5)$.

   (From Unit 3, Lesson 5)

5. The graph at the right shows the line $y=\frac{1}{4}x$.



   a. On the same grid, graph a parallel line that is two units below it.

   b. Write the equation of the new line in slope-intercept form.

   c. Write the y-intercept of the new line as an ordered pair.

   d. Write the x-intercept of the new line as an ordered pair.

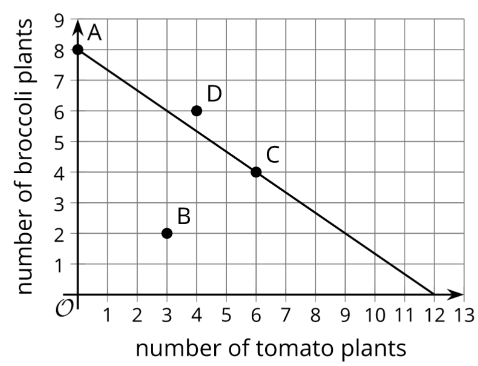
   e. Write the equation of the new line in point-slope form using the y-intercept.

   f. Write the equation of the new line in point-slope form using the x-intercept.

   g. Explain in what ways the equations in parts b, e, and f are the same and in what way they are different.

   (From Unit 3, Lessons 4–5)[4]

6. To grow properly, each tomato plant needs 1.5 square feet of soil and each broccoli plant needs 2.25 square feet of soil. The graph shows the different combinations of broccoli and tomato plants in an 18 square foot plot of soil.[5]



   Match each point to the statement that describes it.

a. Point A	1. The soil is fully used when six tomato plants and four broccoli plants are planted.
b. Point B	2. Only broccoli was planted, but the plot is fully used and all plants can grow properly.
c. Point C	3. After three tomato plants and two broccoli plants were planted, there is still extra space in the plot.
d. Point D	4. With four tomato plants and six broccoli plants planted, the plot is overcrowded.

   (From Unit 3, Lesson 1)

[4] Adapted from Geometry, Module 6, Lesson 6.2 Mathematics Vision project http://www.mathematicsvisionproject.org, licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0)

[5] Adapted from IM 9–12 Math Algebra 1, Unit 2, Lesson 8 https://curriculum.illustrativemathematics.org/HS/index.html, copyright 2019 by Illustrative Mathematics. Licensed under the Creative Commons Attribution 4.0 license https://creativecommons.org/licenses/by/4.0/.

7. A catering company is setting up for a wedding. They expect 150 people to attend. They can provide small tables that seat 6 people and large tables that seat 10 people.[6]

   a. Find a combination of small and large tables that seats exactly 150 people.

   b. Let x represent the number of small tables and y represent the number of large tables. Write an equation to represent the relationship between x and y.

   c. Explain what the point $(20, 5)$ means in this situation.

   d. Is the point $(20, 5)$ a solution to the equation you wrote? Explain your reasoning.

   (From Unit 3, Lesson 1)

8. Indicate if the following statements are true or false. Explain your thinking.[7]

   a. The notation $12 < x$ means the same thing as $x < 12$. It works just like $12 = x$ and $x = 12$.

   b. The inequality $-2(x + 10) \geq 75$ says the same thing as $-2x - 20 \geq 75$. I can multiply by -2 on the left side without reversing the inequality symbol.

   c. When solving the inequality $10x + 22 < 2$, the second step should say $10x > -20$ because I added -22 to both sides and I got a negative number on the right.

   d. When solving the inequality $-5x \geq 45$, the answer is $x \leq -9$.

   e. The words that describe the inequality $x \geq 100$ are "x is greater than or equal to 100."

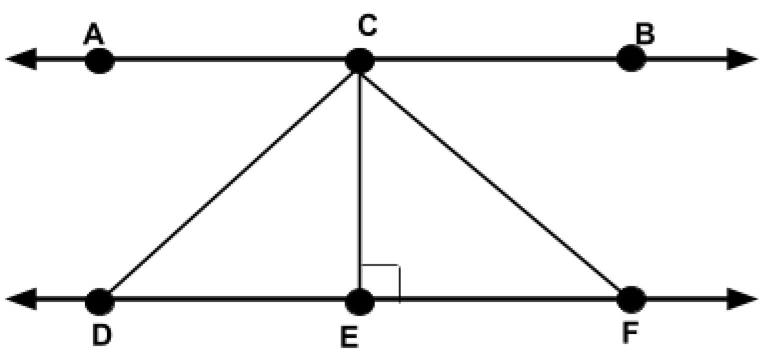
   (From Unit 2)

9. Solve the multi-step inequality: $\frac{3(x-4)}{12} \leq \frac{2x}{3}$

   (From Unit 2)[8]

10.

   a. Which segment below is perpendicular to line AB? How do you know?



   b. Which segment has the shortest length: CD, CE, or CF? How do you know?

   (Addressing NC.4.G.1)

[6] Adapted from IM 9–12 Math Algebra 1, Unit 2, Lesson 9 https://curriculum.illustrativemathematics.org/HS/index.html, copyright 2019 by Illustrative Mathematics. Licensed under the Creative Commons Attribution 4.0 license https://creativecommons.org/licenses/by/4.0/.
[7] Adapted from Algebra 1, Module 4, Lesson 4.4 Mathematics Vision project http://www.mathematicsvisionproject.org, licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0)
[8] Adapted from Algebra 1, Module 4, Lesson 4.4 Mathematics Vision project http://www.mathematicsvisionproject.org (see above).

Lesson 7: Perimeter and Area of Shapes in the Coordinate Plane

PREPARATION

Lesson Goals	Learning Targets
• Given coordinates of a polygon in the coordinate plane, compute the lengths of segments and side lengths of polygons by finding the distance between points in the coordinate plane to calculate: – the perimeter of polygons – the area of triangles and rectangles • Use coordinates to determine a particular type of quadrilateral.	• I can find the length of a side of a polygon in the coordinate plane. • I can find the perimeter and area of triangles and quadrilaterals in the coordinate plane. • I can determine if a quadrilateral in the coordinate plane is a parallelogram, rectangle, rhombus, or square.

Lesson Narrative

In this lesson, students review a method for finding distance between two points in a coordinate plane, familiar to them from middle school: draw a "slope triangle" and use the Pythagorean Theorem. This method is encapsulated as the distance formula.

Students use the technique of finding distance to find the perimeter of various polygons. Students are also asked to find the area of parallelograms and rectangles: for this, finding side lengths may help, but students might prefer to use techniques such as decomposition of shapes that they learned in middle school. (MP1)

Having also just learned criteria for determining whether lines are parallel, perpendicular, or neither, students can now test whether quadrilaterals in the coordinate plane are really squares, rectangles, parallelograms, and rhombuses. They can use these criteria to provide a proof that the shape is what it is purported to be, or explain why the shape does not meet the criteria. In doing so, they construct viable arguments and critique the reasoning of others (MP2).

In what ways will you encourage students to persevere in this lesson?

Focus and Coherence

Building On	Addressing
NC.4.MD.3: Solve problems with area and perimeter. ● Find areas of rectilinear figures with known side lengths. ● Solve problems involving a fixed area and varying perimeters and a fixed perimeter and varying areas. ● Apply the area and perimeter formulas for rectangles in real world and mathematical problems. **NC.5.G.3:** Classify quadrilaterals into categories based on their properties. ● Explain that attributes belonging to a category of quadrilaterals also belong to all subcategories of that category. ● Classify quadrilaterals in a hierarchy based on properties. **NC.8.G.8:** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	**NC.M1.G-GPE.4:** Use coordinates to prove simple geometric theorems algebraically. Use coordinates to solve geometric problems involving polygons algebraically. ● Use coordinates to compute perimeters of polygons and areas of triangles and rectangles. ● Use coordinates to verify algebraically that a given set of points produces a particular type of triangle or quadrilateral.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*10 minutes*)
- **Activity 1** (*10 minutes*)
- **Activity 2** (*10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U3.L7 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

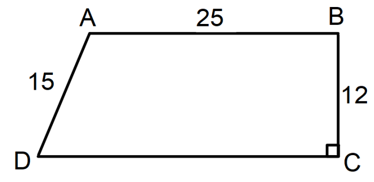
Building On: NC.8.G.8

The purpose of this bridge is for students to use the Pythagorean Theorem to find the unknown side-lengths of a trapezoid in order to determine the area. This problem requires creativity and persistence as students decompose the given trapezoid into other polygons in order to find the length of the missing side. Ideally, students should be thinking in terms of adding auxiliary lines (MP7), but some students may not think of such an approach. If a group of students are struggling unproductively with this task, suggest that the students draw one or both of the altitudes shown in the solution as a way to provide some scaffolding.

Student Task Statement[1]

Quadrilateral $ABCD$ is a trapezoid, $AD = 15$, $AB = 25$, and $BC = 12$.

1. What is the area of the trapezoid?
2. What is the perimeter of the trapezoid?



[1] Adapted from https://tasks.illustrativemathematics.org/

DO THE MATH

PLANNING NOTES

Warm-up: How Do You Know? (*10 minutes*)

Building On: NC.4.MD.3; NC.5.G.3; NC.8.G.8	**Building Towards:** NC.M1.G-GPE.4

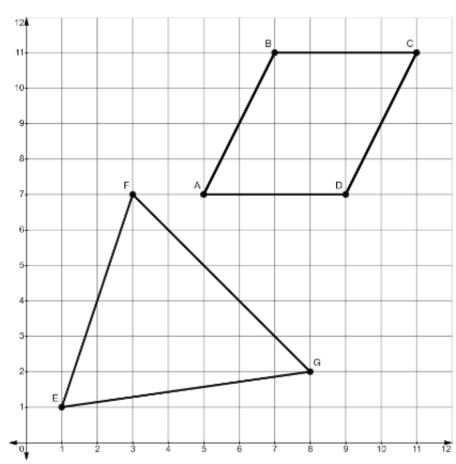
The purpose of this task is for students to activate two pieces of knowledge from middle school: using the Pythagorean Theorem to find lengths in the coordinate plane, and finding areas of polygons by decomposition, “boxing in,” or using altitudes.

Step 1

- Ask students to arrange themselves in groups of two or use visibly random grouping. Before beginning, ask for a student volunteer to explain what a parallelogram is. Make sure all students understand that a parallelogram is a quadrilateral with two pairs of parallel sides.
- Offer 2–3 minutes of quiet think time for students to work independently on strategies and then share what they’ve come up with their partner.
- Partners work together to complete the task.

Student Task Statement

1. Is quadrilateral $ABCD$ a parallelogram? Explain or show your reasoning.
2. What is the area of $ABCD$? Explain or show your reasoning.
3. What is the perimeter of $ABCD$? Round to the nearest hundredth. Explain or show your reasoning.
4. Is triangle EFG isosceles? Explain or show your reasoning.



Step 2

- Facilitate a discussion focused on each question in turn:
  - How do we know whether $ABCD$ is a parallelogram? (The opposite sides are parallel.)
  - How do we know $ABCD$ has parallel sides? (AB and CD both have a slope of 2, while BC and AD both have a slope of 0.)
  - How did you find the area of $ABCD$? (I used the formula $A = bh$ and found the height by counting four units down; I "cut" the parallelogram by drawing a vertical line down from B and moved that piece so that AB lines up with CD to make a square; I counted grid squares and subtracted the extra.)
  - How did you find the perimeter of $ABCD$? (I found the lengths of AD and BC by counting (or subtracting) but used the Pythagorean Theorem to find AB and CD.)
  - Is triangle EFG isosceles? (Yes.) Equilateral? (No.) How did you compare the side lengths of triangle EFG? (I used the Pythagorean Theorem; I started with sides EG and FG because they looked the closest in length.)

DO THE MATH	PLANNING NOTES

Activity 1: Finding the Distance (*10 minutes*)

Addressing: NC.M1.G-GPE.4

The purpose of this task is to provide practice finding the perimeter of triangles to build up to the distance formula. As students calculate distances in the plane, they can see that the process for finding distance involves similar steps each time. These steps can be encapsulated in the distance formula, which is developed in Step 2 of this activity. Going forward, students can use the distance formula to find lengths if they like, or they can choose to continue the approach of adding lines and using the Pythagorean Theorem.

Step 1

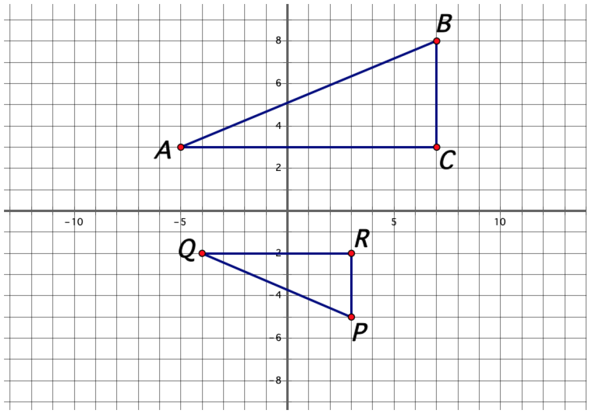
- Keep students in the same groups and give them 2 minutes of quiet work time to complete question 1. Then have students compare answers to question 1 and begin question 2.

Monitoring Tip: Pay attention to the way students are finding the lengths of the vertical and horizontal sides. Some may simply count, some may use subtraction, and for side AC, some may find the length on each side of the *y*-axis and add to find the total length. Take note of students who use subtraction, especially for side AC.

Advancing Student Thinking: For question 2, students may not understand what it means to find lengths in terms of the given coordinates. Reassure these students that the answers will not be numbers. Ask them to describe what they did to find the length of BC in question 1. What are the *y*-coordinates in this new situation, and how can we write the distance between them?

Student Task Statement

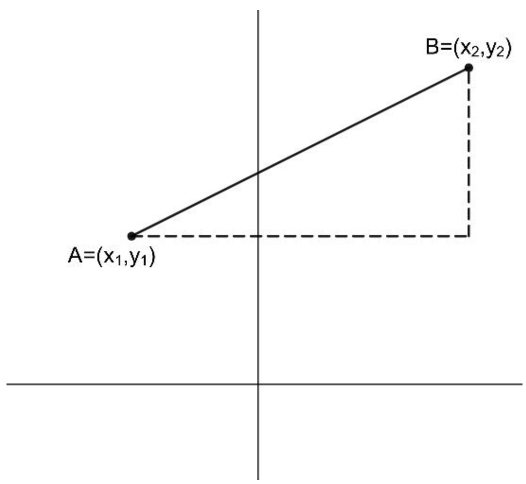
1. Below is a picture of two triangles with vertices on coordinate grid points[2]:



What are the perimeters of ΔABC and ΔPQR?

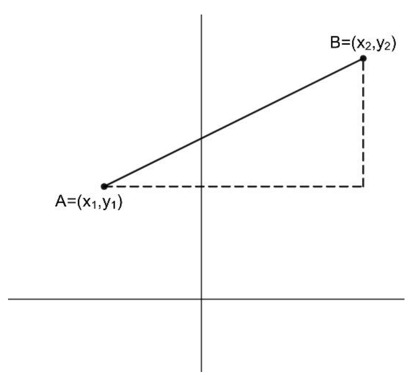
2. Here is a line segment that can stand for any line segment. Because its coordinates can be any values, we can call the coordinates of A (x_1, y_1) and the coordinates of B (x_2, y_2).

   Find the lengths of the two dashed lines in terms of those coordinates.



Are You Ready For More?

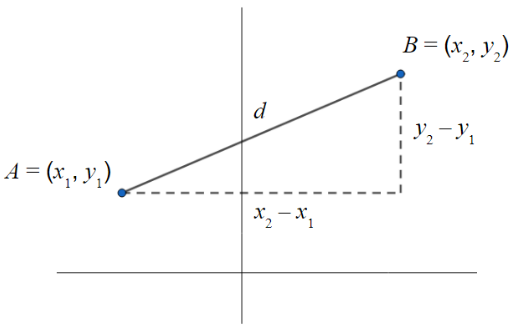
Using the same graph above, find the length of AB in terms of x_1, x_2, y_1, and y_2.



[2] Adapted from https://tasks.illustrativemathematics.org/

Step 2

- When all groups have had a chance to think about (but not necessarily complete) the second question, display the diagram in the second question for all to see. Facilitate a whole-class discussion.
  - Ask: "What are the lengths of each dashed line? How do you know?" If needed, call upon students who used subtraction to find the length of AC in the first question to explain why the horizontal length is $x_2 - x_1$. (You subtract the coordinates, because that is how you find distance on a number line; for triangle ABC, the vertical distance was 5 because that's 8 minus 3.) Annotate the diagram with the lengths $x_2 - x_1$, $y_2 - y_1$, and d, like so:



  - Tell students that there is a formula for finding distance which is a way of writing down the steps they have already been doing. This diagram is used to find the formula.
  - Ask students how we can find the length d, now that we know the horizontal and vertical sides. Once students agree that the Pythagorean Theorem is needed, write $(x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2$ and ask if that must be true.
  - Finally, ask students what we can do to solve for d. (Take the square root of each side.) Write the final formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d$

DO THE MATH	PLANNING NOTES

Activity 2: What Shape Am I? (*10 minutes*)

Instructional Routines: Round Robin; Discussion Support (MLR8) - Responsive Strategy	
Addressing: NC.M1.G-GPE.4	

The purpose of this activity is for students to connect several of the ideas they have learned about coordinate geometry. In proving whether shapes are - or are not - what they are purported to be, they must check slopes to determine whether lines are parallel/perpendicular and calculate distances to determine whether side lengths are equal. Finally, they calculate the area of a rectangle, perhaps using a new strategy available to them: multiplying the length and width of the sides. In order to decide which calculations to do, students must think carefully about the definitions of each shape. Both the Round Robin routine and the activity debrief provide opportunities to clarify these definitions.

RESPONSIVE STRATEGY

Use this routine to help students produce statements about their reasoning. Provide sentence frames for students to use when sharing such as, "My quadrilateral is/isn't a __ because it has ___."

Discussion Supports (MLR8)

Step 1

- Ask students to arrange themselves in groups of four or use visibly random grouping.
- In each group, students assign themselves questions 1–4, so that as a group, each question will be answered.
  - Provide students 3–4 minutes to solve the question and then, using the *Round Robin* routine, ask students to share their answer and their reasoning.

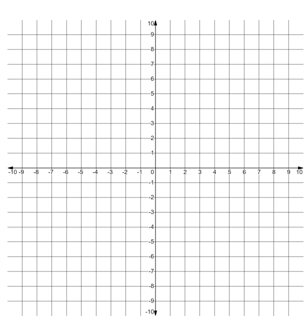
Step 2

- After all students have had a chance to share, students work together in their groups to solve question 5.

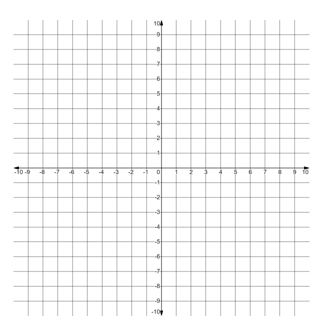
Advancing Student Thinking: In question 1, students may think that it is enough to check that four sides are equal in length for it to be a square. Show these students that there is more to do by drawing a picture of a rhombus. Likewise, some students may think that it is enough to check for four right angles. For these students, draw a rectangle.

Student Task Statement

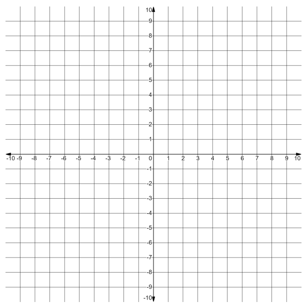
1. A quadrilateral has vertices $A(1,0)$, $B(5,-2)$, $C(7,2)$, and $D(3,4)$. Is $ABCD$ a square?



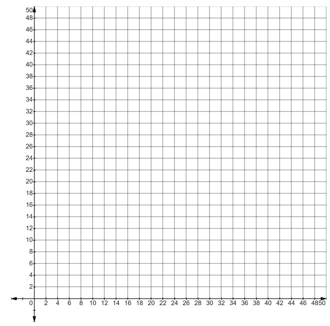
2. A quadrilateral has vertices $E(-2,3)$, $F(-5,11)$, $G(1,9)$, and $H(4,1)$. Is $EFGH$ a parallelogram?



3. A quadrilateral has vertices $K(-5,-1)$, $L(-1,-4)$, $M(5,4)$, and $N(1,7)$. Is $KLMN$ a rectangle?



4. A quadrilateral has vertices $Q(1,10)$, $R(26,33)$, $S(50,13)$, $T(26,20)$. Is $QRST$ a rhombus?



5. Find the area of the quadrilateral in question 3.

Step 3

- Facilitate a whole-class discussion focused on what tests need to be done for each shape. Discussion of the last question will be saved for the Lesson Debrief.
  - Which question required checking slopes? What were you looking for when you checked them? (In questions 1 (square) and 3 (rectangle), we needed to check that the slopes were opposite reciprocals. In question 2 (parallelogram), we needed to check that the pairs of opposite sides had the same slopes.)
  - Which questions required calculating lengths? (In questions 1 (square) and 4 (rhombus), we had to check that all four sides were equal.)
    - Some students may have approached question 3 (parallelogram) by showing that opposite pairs of sides were the same length. While this strategy does work, it does not use the definition of a parallelogram.
  - Rectangles have to have equal pairs of opposite sides. Should we have found the side lengths of the rectangle to show that it was really a rectangle? (No, having four right angles automatically makes opposite side lengths the same.)

DO THE MATH	**PLANNING NOTES**

Lesson Debrief (*5 minutes*)

In this lesson, students used their understanding of parallel and perpendicular lines and of distance to examine quadrilaterals and other polygons. They used these ideas to check whether quadrilaterals were a certain shape, and they found the area and perimeter of polygons. Here, students will use questions 3 and 5 from the previous activity to review and extend these ideas.

Display the rectangle from question 3 of the previous activity. Ask students:

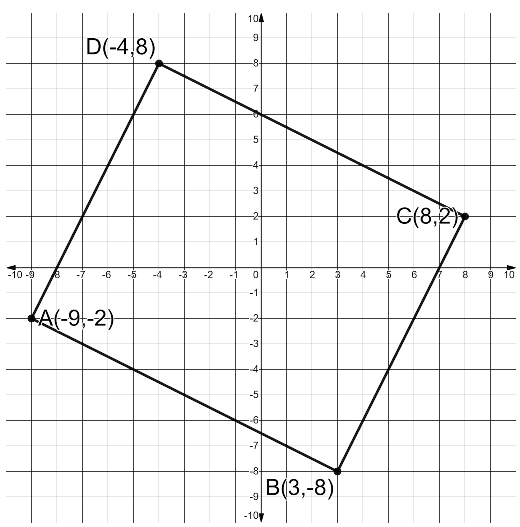
- "How did you find the area of the rectangle?" (I drew a box around the rectangle and subtracted the "extra" area of the triangles created; I calculated the length and width and multiplied them together.)
- Take students through the method of finding the side lengths and multiplying them together, bringing it up if no one else did:
  - "What are some ways to find the length of each side of the rectangle?" (Use the Pythagorean theorem or the distance formula.)
  - "Why does it work to find the area of the shape by multiplying the side lengths together?" (The shape is a rectangle, and the formula for the area of a rectangle is length $\times$ width.)
  - "Would this work for finding the area of a parallelogram?" (No, because the height of a parallelogram is not (usually) a side length.)
- "How can we now quickly find the perimeter of the rectangle?" (We already calculated the lengths of the sides, so now we can add them together.)

PLANNING NOTES

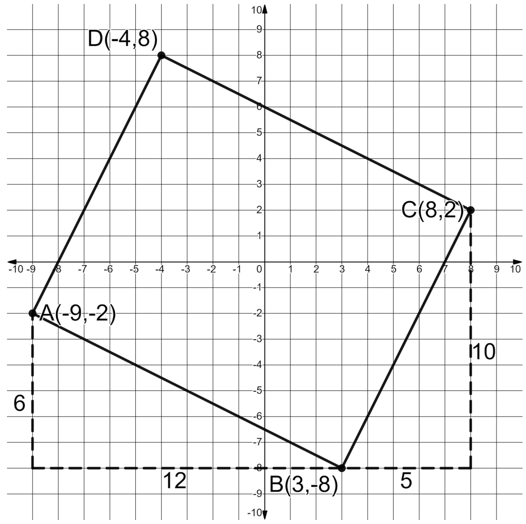
Student Lesson Summary and Glossary

Take a look at quadrilateral $ABCD$. Is it a square? Could be, but it looks a little squished. A rectangle? Maybe the corners aren't perfect right angles. Or maybe a parallelogram? Or none of these shapes?

We can use ideas about distance, parallel lines, and perpendicular lines to find out.



To be a square, $ABCD$ would have to have all of its sides the same length, as well as having all 90-degree angles. Let's start by checking the side lengths. To find the lengths of AB and BC, draw in extra lines so that we can use the Pythagorean Theorem:



$$AB = \sqrt{6^2 + 12^2}$$
$$AB = \sqrt{180} \approx 13.42$$

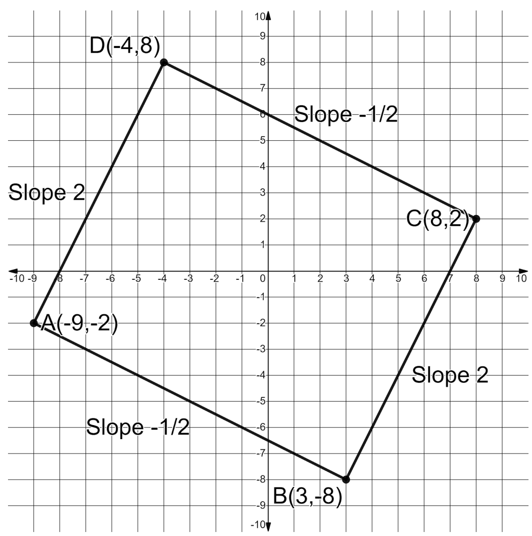
$$BC = \sqrt{5^2 + 10^2}$$
$$BC = \sqrt{125} \approx 11.18$$

Since AB and BC are different lengths, $ABCD$ can't be a square. But it could still be a rectangle. Let's check the slopes now to see if the sides are perpendicular. We've already made slope triangles for sides AB and BC, so let's start there.

Slope of AB: $-\frac{6}{12} = -\frac{1}{2}$, Slope of BC: $\frac{10}{5} = \frac{2}{1}$

So far so good: $-\frac{1}{2}$ and 2 are opposite reciprocals. But that only tells us that angle B is a right angle; we don't know about the others. We should find the slopes of the other sides. $-\frac{1}{2}$ and 2 are opposite reciprocals. But that only tells us that angle B is a right angle; we don't know about the others. We should find the slopes of the other sides.

Slope of CD: $\frac{2-8}{8-(-4)} = -\frac{6}{12} = -\frac{1}{2}$, Slope of DA: $\frac{8-(-2)}{-4-(-9)} = \frac{10}{5} = \frac{2}{1}$



Since each angle is formed by a pair of line segments, one with slope $\frac{1}{2}$ and the other with slope 2, all four angles are right angles. We can now say for sure that $\frac{1}{2}$ and the other with slope 2 $ABCD$ is a rectangle.

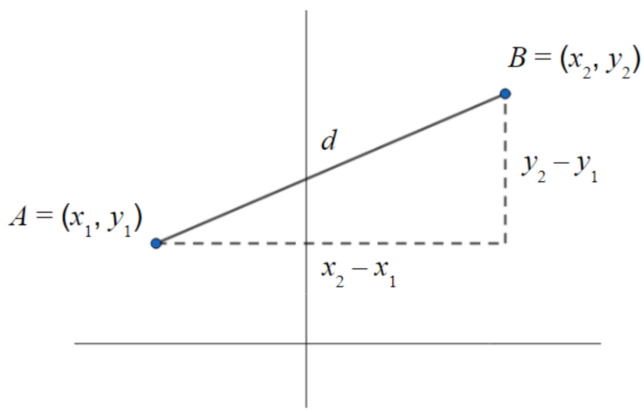
This also shows that $ABCD$ must be a parallelogram. Each pair of opposite sides has the same slope, which means that each pair of opposite sides is parallel.

Now that we know that $ABCD$ is a rectangle, we can find its area and perimeter.

We've already found two of its side lengths, and since it is a rectangle the other two side lengths must match. So the perimeter of $ABCD$ is $2 \cdot \sqrt{180} + 2 \cdot \sqrt{125} \approx 49.19$.

To find the area of $ABCD$, we can multiply the length times the width: $\sqrt{180} \cdot \sqrt{125} \approx 150$.

When we calculated the side lengths AB and CD, we made a triangle with horizontal and vertical sides and used the Pythagorean theorem to find the length of the hypotenuse. Here is a more general picture of what we did:



To find the length of the horizontal sides, subtract the x-coordinates. To find the length of the vertical sides, subtract the y-coordinates. When we use the Pythagorean theorem on these side lengths, we get the **distance formula**.

Distance formula: The distance d between points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

When dealing with small numbers, you will never need to use the distance formula as long as you remember to use the Pythagorean theorem to find lengths. However, the distance formula will come up in more advanced courses.

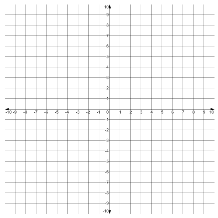
Cool-down: What Kind of Quad are You? (*5 minutes*)

Addressing: NC.M1.G-GPE.4

Cool-down Guidance: More Chances
If students need additional practice with the skills and understandings in this cool-down, practice problems in future lessons can be assigned, as well as an opportunity for additional support in Lessons 13 & 14 teacher-led small-group instruction.

Cool-down

1. A quadrilateral has vertices $A = (0,1)$, $B = (2,4)$, $C = (0,5)$, and $D = (-2,2)$.



   a. Use the coordinate grid to sketch quadrilateral $ABCD$.

   b. Does quadrilateral $ABCD$ have two pairs of parallel sides? Show how you know.

   c. Does quadrilateral $ABCD$ have four right angles? Show how you know.

   d. Find the perimeter of quadrilateral $ABCD$.

Student Reflection: I love math most when ______________ and I do not like it when _____________________________.

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

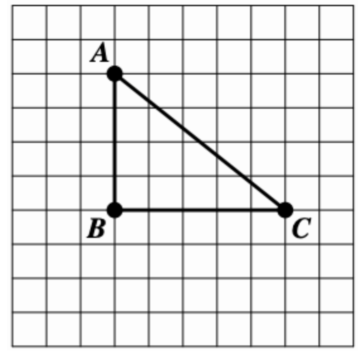
TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Reflect on how comfortable your students are asking questions of you and of each other. What can you do to encourage students to ask questions?

Practice Problems

1. Use the graph.[3]



   a. Find AB.

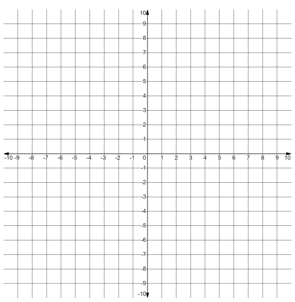
   b. Find BC.

   c. Find AC.

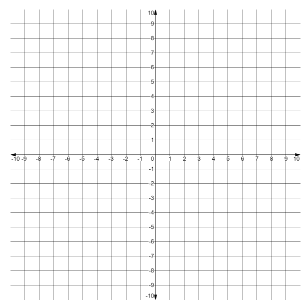
   d. Why is it easier to find the distance between point A and point B and point C than it is to find the distance between point A and point C?

   e. Explain how to find the distance between point A and point C.

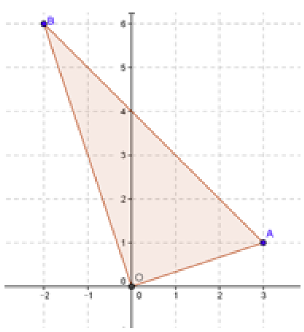
2. A quadrilateral has vertices $A=(0,0)$, $B=(1,3)$, $C=(0,4)$, and $D=(-1,1)$. Is $ABCD$ a parallelogram? How do you know?



3. A quadrilateral has vertices $A=(0,0)$, $B=(2,4)$, $C=(0,5)$, and $D=(-2,1)$. Is $ABCD$ a rectangle? How do you know?

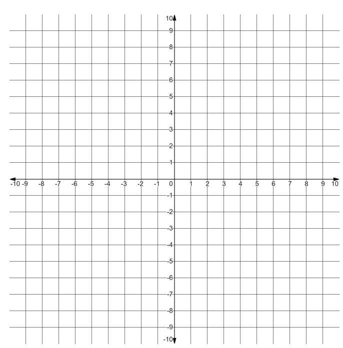


4. Given the points $O\,(0,0)$, $A\,(3,1)$, and $B\,(-2,6)$, prove OA is perpendicular to OB.



(From Unit 3, Lesson 6)[4]

5. Han thinks that the points $(4,2)$ and $(-1,4)$ form a line perpendicular to a line with slope 4. Do you agree? Why or why not?[5]



(From Unit 3, Lesson 6)

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[3] Adapted from Secondary Math 1, Module 8, Lesson 1 Mathematics Vision project http://www.mathematicsvisionproject.org, licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0)
[4] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds. It is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States (CC BY-NC-SA 3.0 US).
[5] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education (see above).

6. Write the equation of the line passing through $(-3, 4)$ and perpendicular to $-2x + 7y = -3$.

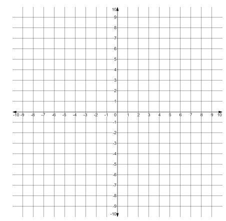
   (From Unit 3, Lesson 6)[6]

7. Are the pairs of lines parallel, perpendicular or neither? Explain your reasoning.[7]

   a. $3x + 2y = 74$ and $9x - 6y = 15$

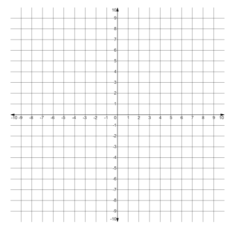
   b. $4x - 9y = 8$ and $18x + 8y = 7$

   (From Unit 3, Lessons 5 and 6)



8. Write an equation of the line that passes through $(2, 4)$ and has a slope of -3.

   (From Unit 3, Lesson 4)



9. Consider the equation $2.5x + 5y = 20$. For each question, explain or show your reasoning.

   a. If we graph the equation, what is the slope of the graph?

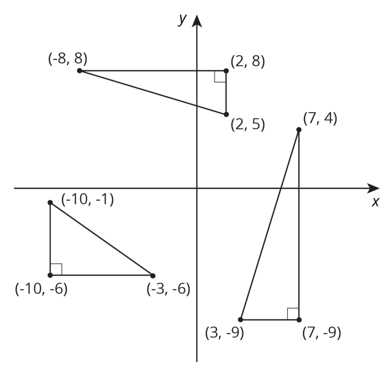
   b. Where does the graph intersect the y-axis?

   c. Where does it intersect the x-axis?

   (From Unit 3, Lesson 3)

10. The right triangles are drawn in the coordinate plane, and the coordinates of their vertices are labeled. For each right triangle, label each leg with its length.[8]

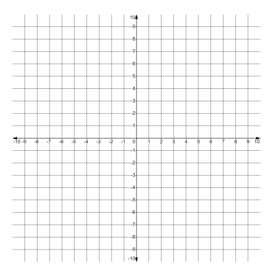
    (Addressing NC.8.G.8)



11. Find the distance between each pair of points. If you get stuck, try plotting the points.[9]

    a. $M = (0, -11)$ and $P = (0, 2)$
    b. $A = (0, 0)$ and $B = (-3, -4)$
    c. $C = (8, 0)$ and $D = (0, -6)$

    (Addressing NC.8.G.8)



[6] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds. It is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States (CC BY-NC-SA 3.0 US).
[7] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education (see above).
[8] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).
[9] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html (see above).

Lesson 8: Writing and Graphing Systems of Linear Equations

PREPARATION

Lesson Goals	Learning Targets
• Solve systems of linear equations by reasoning with tables and by graphing, and explain (orally and in writing) the solution method. • Understand that the solution to a system of equations in two variables is a pair of values that simultaneously make both equations true, and that it is represented by the intersection point of the graphs of the equations. • Understand that two (or more) equations that represent the constraints on the same quantities in the same situation form a system.	• I can use tables and graphs to solve systems of equations • I can explain what we mean by "the solution to a system of linear equations" and can explain how the solution is represented graphically. • I can explain what we mean when we refer to two equations as a system of equations.

Lesson Narrative

This is the first of a series of lessons in which students review what they learned about **systems of equations** in middle school and develop new techniques for solving them.

In this lesson, students recall that a system of equations in two variables is a set of equations that represent multiple constraints in the same situation, and that a **solution to the system** is any pair of values that satisfy all the constraints simultaneously. Students also revisit the idea that the solution, if there is one, can be represented graphically as the intersection of the graphs of the equations.

Students write a system of equations to represent the quantities and constraints in each of several situations, find a solution that meets multiple constraints by graphing, and then interpret the solution in context. In the process, they reason both abstractly and quantitatively (MP2). As they analyze relationships mathematically and reflect on the results, students also engage in aspects of modeling (MP4).

Some students may attempt to solve the systems algebraically. This is appropriate and welcome, but it is not necessary to introduce the idea to the class here. Students will use algebra to solve systems starting in the next lesson.

What is the main purpose of this lesson? What is the one thing you want your students to take away from this lesson? How might you differentiate your instruction to support students with diverse needs?

Focus and Coherence

Building On	Addressing	Building Towards
NC.8.EE.8: Analyze and solve a system of two linear equations in two variables in slope-intercept form. • Understand that solutions to a system of two linear equations correspond to the points of intersection of their graphs because the point of intersection satisfies both equations simultaneously. • Solve real-world and mathematical problems leading to systems of linear equations by graphing the equations. Solve simple cases by inspection**.**	**NC.M1.A-CED.3:** Create systems of linear equations and inequalities to model situations in context. **NC.M1.A-REI.6:** Use tables, graphs, or algebraic methods (substitution and elimination) to find approximate or exact solutions to systems of linear equations and interpret solutions in terms of a context.	**NC.M1.A-REI.10:** Understand that the graph of a two-variable equation represents the set of all solutions to the equation.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*20 minutes*)
- **Activity 2** (*5 minutes*)
  - Technology is required for this lesson: Acquire devices that can access Desmos (recommended) or other graphing technology. It is ideal if each student has their own device.
  - Blank visual displays for each student group (possible visual display options: poster board, chart paper, Google Slides, Jamboard)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U3.L8 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional*, *5 minutes*)

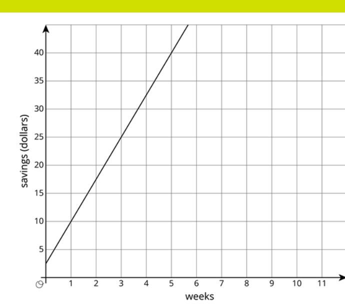
Building From: NC.8.EE.8

The purpose of this bridge is to elicit the idea that the intersection point of two graphs is a solution to each of the equations. In this case, x represents the number of weeks in which Noah and Andre have saved the amount of money, and y represents that amount of money. This idea will be useful when students solve systems of equations graphically later in the lesson. While students may notice and wonder many things about these images, intersection points and solutions are the important discussion points. This task is aligned to question 3 in Check Your Readiness.

Student Task Statement

Andre and Noah started tracking their savings at the same time. Andre started with $15 and deposits $5 per week. Noah started with $2.50 and deposits $7.50 per week. The graph of Noah's savings is given and his equation is $y = 7.5x + 2.5$, where x represents the number of weeks and represents his savings.[1]

Write the equation for Andre's savings and graph it alongside Noah's. What does the intersection point mean in this situation?



[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

DO THE MATH	PLANNING NOTES

Warm-up: A Possible Mix? (*5 minutes*)

Instructional Routines: Math Talk; Discussion Supports (MLR8) - Responsive Strategy
Building Towards: NC.M1.A-REI.6

This *Math Talk* refreshes students' knowledge about constraints and the values that meet them. Students are prompted to determine if four given combinations of raisins and walnuts meet a certain cost constraint. The reasoning elicited here prepares them to write and solve systems of linear equations in two variables later.

Many students are likely to approach the task by multiplying pounds of raisins by 4 and pounds of walnuts by 8, and then see if the sum of the two products is 15. Some students may arrive at their conclusions by reasoning and estimating. For example, they may reason that the last combination is impossible because at $4 per pound, 3.5 pounds of raisins would cost more than $12, so it is not possible to also get 1 pound of walnuts and pay only $15 total.

To explain their reasoning, students need to be precise in their word choice and use of language (MP6).

MATH TALK	***What Is This Routine?*** In these warm-ups, one problem is displayed at a time. Students are given a few moments to quietly think and give a signal when they have an answer and a strategy. The teacher selects students to share different strategies for each problem, asking, "Who thought about it a different way?" Their explanations are recorded for all to see. Students might be pressed to provide more details about why they decided to approach a problem a certain way. It may not be possible to share every possible strategy in the given time—the teacher may only gather two or three distinctive strategies per problem. Problems are purposefully chosen to elicit different approaches, often in a way that builds from one problem to the next. ***Why This Routine?*** A *Math Talk* builds fluency by encouraging students to think about the numbers, shapes, or algebraic expressions and rely on what they know about structure, patterns, and properties of operations to mentally solve a problem. While participating in these activities, there is a natural need for students to be precise in their word choice and use of language (MP6). Additionally, a Math Talk often provides opportunities to notice and make use of structure (MP7).

Step 1

- Display one problem at a time.
- Give students quiet think time for each problem and ask them to give a signal when they have an answer and a strategy.
- Keep all problems displayed throughout the talk.

RESPONSIVE STRATEGY

To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Student Task Statement

Diego bought some raisins and walnuts to make trail mix. Raisins cost $4 per pound, and walnuts cost $8 per pound. Diego spent $15 on both ingredients.

Decide if each pair of values could be a combination of raisins and walnuts that Diego bought.

1. 1.5 pounds of raisins and 1 pound of walnuts
2. 1 pound of raisins and 1.5 pounds of walnuts
3. 2.25 pounds of raisins and 0.75 pounds of walnuts
4. 3.5 pounds of raisins and 1 pound of walnuts

Step 2

- Facilitate the *Math Talk* routine by asking students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:
  - “Who can restate ___’s reasoning in a different way?”
  - “Did anyone have the same strategy but would explain it differently?”
  - “Did anyone solve the problem in a different way?”
  - “Does anyone want to add on to ___’s strategy?”
  - “Do you agree or disagree? Why?”
- Remind students that the given situation involves a cost constraint. Emphasize that only the third option, 2.25 pounds of raisins and 0.75 pounds of walnuts, satisfies the constraint. One way to check if certain values meet the constraint is by writing an equation and checking if it is true. For example, the equation $4(2.25) + 8(0.75) = 15$ is true. If we replaced the weights of raisins and walnuts with other pairs of values, the equation would be false.

RESPONSIVE STRATEGY

Display sentence frames to support students when they explain their strategy. For example, “First, I ___ because....” or “I noticed ___ so I....” Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Discussion Supports (MLR8)

DO THE MATH

PLANNING NOTES

Activity 1: Trail Mix (*20 minutes*)

Building On: NC.8.EE.8	**Addressing:** NC.M1.A-CED.3; NC.M1.A-REI.6	**Building Towards:** NC.M1.A-REI.10

In this activity, students write and graph equations to represent two constraints in the same situation and use tables and graphs to see possible values that satisfy the constraints. The work prompts them to think about a pair of values that simultaneously meets multiple constraints in the situation, which in turn helps them make sense of the phrase "a solution to both equations."

Step 1

- Explain to students that they will now examine the same situation involving two quantities (raisins and walnuts), but there are now two constraints (cost and weight).
- Briefly remind students how to create a graph using graphing technology, such as Desmos, if needed.
- Provide students 3–4 minutes of quiet time to complete the first set of questions about the cost constraint.

M

Monitoring Tip: Monitor for these likely strategies for completing the tables, from less precise to more precise:
- guessing and checking
- using the graph (by identifying the point with a given x- or y-value and estimating the unknown value, or using technology to get an estimate)
- substituting the given value of one variable into the equation and solving for the other variable

Identify students who use each strategy and let them know that they may be asked to share later. Intentionally seek to highlight approaches from students who do not typically volunteer.

Step 2

- Pause for a whole-class discussion.
- Select previously identified students to share their strategies, in the order listed in the Monitoring Tip. If one of the strategies is not mentioned, bring it up.
- Ask a student who used the graph to complete the table to explain how exactly the graph was used. For example: "How did you use the graph of $4x + 8y = 15$ to find y when x is 2?" Make sure students recognize that this typically involves clicking and dragging over different points on the graph or tracing the graph to get the coordinates.

RESPONSIVE STRATEGY

Provide appropriate reading accommodations and supports to ensure student access to written directions, word problems and other text-based content.

Supports accessibility for: Language; Conceptual processing

Step 3

- Ask students to arrange themselves in pairs or use visibly random grouping to proceed with the remainder of the activity.
- If time permits, pause again after the second set of questions (about the weight constraint) to discuss how the strategies for completing the second table are like or unlike that of completing the first table. Alternatively, give students a minute to confer with their partner before moving on to the last question.

Advancing Student Thinking: Some students might think that the values in the second table need to reflect a total cost of $15. Clarify that the table represents only the constraint that Diego bought a total of two pounds of raisins and walnuts. Some students may struggle with writing the equation for the second table since the coefficients of x and y are 1. This is very common as students tend to forget that x and y are numbers. In these cases, ask students to think about possible numbers first and then write the equation.

The idea of finding an (x, y) pair that satisfies multiple constraints should be familiar from middle school. If students struggle to answer the last question, ask them to study the values in the table. Ask questions such as: "If Diego bought 0.25 pound of raisins, would he meet both the cost and weight requirements?" and "Which combinations of raisins and walnuts would allow him to meet both requirements? How many combinations are there?"

Student Task Statement

1. Here is a situation you saw earlier: Diego bought some raisins and walnuts to make trail mix. Raisins cost \$4 per pound and walnuts cost \$8 per pound. Diego spent \$15 on both ingredients.

    a. Write an equation to represent this constraint. Let x be the pounds of raisins and y be the pounds of walnuts.

    b. Use graphing technology to graph the equation.

    c. Complete the table with the amount of one ingredient Diego could have bought given the other. Be prepared to explain or show your reasoning.

Raisins (pounds)	Walnuts (pounds)
0	
0.25	
	1.375
	1.25
1.75	
3	

2. Here is a new piece of information: Diego bought a total of 2 pounds of raisins and walnuts combined.

    a. Write an equation to represent this new constraint. Let x be the pounds of raisins and y be the pounds of walnuts.

    b. Use graphing technology to graph the equation.

    c. Complete the table with the amount of one ingredient Diego could have bought given the other. Be prepared to explain or show your reasoning.

Raisins (pounds)	Walnuts (pounds)
0	
0.25	
	1.375
	1.25
1.75	
3	

3. Diego spent \$15 and bought exactly 2 pounds of raisins and walnuts. How many pounds of each did he buy? Explain or show how you know.

Step 4

- Invite students to share their response and reasoning for the last question. If no one volunteers a solution using the tables, ask students to turn to a partner and share how their solution shows up in the two tables they completed.
- Display the graphs representing the system (either created by a student or as shown as sample student response in the Answer Key). Discuss with students:
  - "Can you find other combinations of raisins and walnuts, besides 0.25 pound and 1.75 pounds, that meet both cost and weight constraints?" (No)
  - "How many possible combinations of raisins and walnuts meet both constraints? How do we know?" (One combination, because the graphs intersect only at one point.)
- Explain to students that the two equations written to represent the constraints form a system of equations. A curly bracket is used to indicate a system, like this:

$$\begin{cases} 4x + 8y = 15 \\ x + y = 2 \end{cases}$$

- Highlight that the solution to the system is a pair of values (in this case, pounds of raisins and walnuts) that meet both constraints. This means the pair of values is a solution to both equations. Graphing is an effective way to see the solution to both equations, if one exists.

DO THE MATH	PLANNING NOTES

Activity 2: Meeting Constraints (*5 minutes*)

Instructional Routine: Compare and Connect (MLR7)	
Building On: 8.EE.C.8.b	**Addressing:** NC.M1.A-CED.3; NC.M1.A-REI.6

This activity allows students to apply what they learned about meeting multiple constraints simultaneously and to practice solving simple systems of equations in context.

Making graphing technology available but not suggesting it gives students an opportunity to choose appropriate tools strategically (MP5).

Step 1

- Keep students in pairs.
- Tell students that they will now look at situations that also involve two quantities and two constraints. Their job is to represent the pair of constraints by writing a system of equations, and then to find a solution to the system.

- Assign question 1 to half of the groups and question 2 to the other half of the groups.
- Instruct students to create a visual display of their work as they solve their assigned problem with their partner.

> **Monitoring Tip:** As students work, notice the strategies they use. Many students may choose to graph the systems because that strategy was used in the first activity, but some students may choose to solve the systems by guessing and checking, creating tables, or by reasoning algebraically.

Advancing Student Thinking: If students struggle to sort out the information in a problem, suggest that they start by drawing a picture or diagram to help understand the situation. Another idea is to start by creating one or more tables to list possible combinations of values and observe the relationship between the values.

For students who need more scaffolding, ask questions such as “What equation can you write to say that the number of tables, long tables and round ones, is 25?” and “What do your variables represent?” Then, ask students to use the same variables to write an equation that says that there are 190 seats.

Student Task Statement

Here are two situations that each relate two quantities and involve two constraints. For each situation, find the pair of values that meet both constraints and explain or show your reasoning.

1. A dining hall had a total of 25 tables—some long rectangular tables and some round ones. Long tables can seat eight people. Round tables can seat six people. On a busy evening, all 190 seats at the tables are occupied.

   How many long tables, x, and how many round tables, y, are there?

2. A family bought a total of 16 adult and child tickets to a magic show. Adult tickets are $10.50 each and child tickets are $7.50 each. The family paid a total of $141.

   How many adult tickets, a, and child tickets, c, did they buy?

Are You Ready For More?

1. Make up equations for two lines that intersect at $(4,1)$.
2. Make up equations for three lines whose intersection points form a triangle with vertices at $(-4,0)$, $(2,9)$, and $(6,5)$.

Step 2

- Use the *Compare and Connect* routine by first selecting two pairs of students who used different strategies to set up and solve their system. Make each pairs’ work visible to the class, and ask students to observe what is similar and what is different about each strategy. Prompt discussion by asking, “Where is the total amount shown in each strategy? How were the different constraints handled in each strategy? What is different and what is the same about how the systems were solved in each case?”
- If no students share an algebraic strategy, ask if anyone solved the problems without graphing. If so, invite them to share their rationale. Otherwise, it is not crucial to probe further at this moment.

DO THE MATH

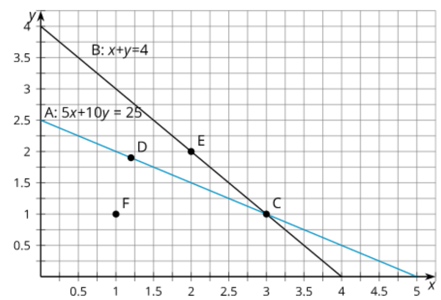
PLANNING NOTES

Lesson Debrief (*5 minutes*)

In the lesson, students reviewed what a system of equations is and how to solve them. They graphed equations using technology and created systems of equations to represent the constraints of a context. Facilitate a discussion using the following questions. As students respond, make connections to examples from the lesson.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

- “Reflect on the strategies that were used today. Some of them—graphing and algebraic solving—required writing equations. Others—guessing and checking, or using tables—did not. Why might it be helpful to write a pair of equations to solve these problems? Are there situations in which it might not be helpful to write equations?”
- "How would you explain ‘a system of equations’ to a classmate who is absent today? What would you say if they asked, ‘What does it mean to solve a system of equations?’”
- "Here are graphs that represent this system $\begin{cases} x+y=4 \\ 5x+10y=25 \end{cases}$."



  - "Which point or points are solutions to the equation $x+y=4$?" (C and E)
  - "Which are solutions to $5x+10y=25$?" (C and D)
  - “Which are solutions to the system?" (C)
- "What is the same and what is different about solving a single linear equation in two variables, say $x+y=4$, and solving a system of equations, say $\begin{cases} x+y=4 \\ 5x+10y=25 \end{cases}$?"

  (Consider setting up a two-column organizer and recording students' responses accordingly. Here is an example.)

PLANNING NOTES

Alike:	**Different:**
• The solutions are pairs of values. • The solutions are points on the graph of each equation. • We can solve by using the graph. • We can solve by substituting different values for x and y and seeing which pair of values make the equation true.	• To solve a linear equation in two variables is to find pairs of values that make one equation true (or meet one constraint). To solve a system is to find pairs of values that simultaneously make both equations in the system true (or meet both constraints in a situation). • There are many solutions to a linear equation in two variables, but there might only be one (or no) solution to a system of linear equations in two variables. • A solution to a linear equation in two variables could be any point on the graph. The solution to a system must be the intersection of the two graphs.

Student Lesson Summary and Glossary

A costume designer needs some silver and gold thread for the costumes for a school play. She needs a total of 240 yards. At a store that sells thread by the yard, silver thread costs $\$0.04$ a yard and gold thread costs $\$0.07$ a yard. The designer has $\$15$ to spend on the thread.

How many of each color should she get if she is buying exactly what is needed and spending all of her budget?

This situation involves two quantities and two constraints—length and cost. Answering the question means finding a pair of values that meets both constraints simultaneously. To do so, we can write two equations and graph them on the same coordinate plane.

Let x represents yards of silver thread and y yards of gold thread.

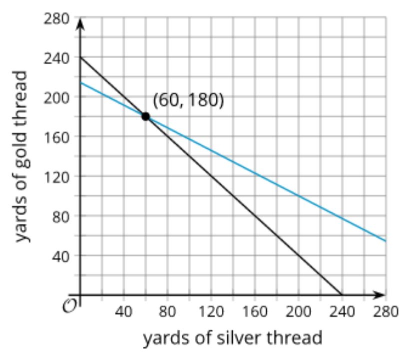
- The length constraint: $x+y=240$
- The cost constraint: $0.04x+0.07y=15$

Every point on the graph of $x+y=240$ is a pair of values that meets the length constraint.

Every point on the graph of $0.04x+0.07y=15$ is a pair of values that meets the cost constraint.

The point where the two graphs intersect gives the pair of values that meets *both* constraints.

That point is $(60,180)$, which represents 60 yards of silver thread and 180 yards of gold thread.



If we substitute 60 for x and 180 for y in each equation, we find that these values make the equation true. $(60,180)$ is a solution to both equations simultaneously.

$$\begin{aligned} x+y&=240\\ 60+180&=240\\ 240&=240 \end{aligned}$$

$$\begin{aligned} 0.04x+0.07y&=15\\ 0.04(60)+0.07(180)&=15\\ 2.40+12.60&=15\\ 15&=15 \end{aligned}$$

The equations we used to solve this problem are known as a **system of equations.**

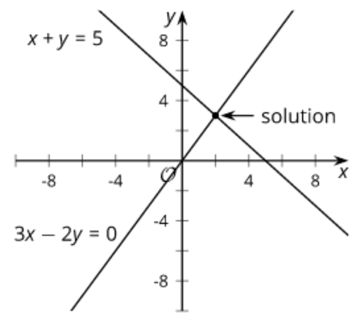
System of equations: Two or more equations that represent the constraints in the same situation form a system of equations.

A curly bracket is often used to indicate a system: $\begin{cases} x+y=240 \\ 0.04x+0.07y=15 \end{cases}$

We say that this system of equations has the **solution** $(60, 180)$.

Solution to a system of equations: A pair of values that makes all of the equations in the system true.

Graphing the equations is one way to find the solution to a system of equations. On the graph shown, the solution is the point where the graphs intersect.



Cool-down: Fabric Sale (*5 minutes*)

Addressing: NC.M1.A-REI.6

Cool-down Guidance: More Chances
The cool-down of Lesson 11 gives another opportunity for students to discuss the meaning of a solution in context.

Cool-down

At a fabric store, fabrics are sold by the yard. A dressmaker spent $\$36.35$ on 4.25 yards of silk and cotton fabrics for a dress. Silk is $\$16.90$ per yard and cotton is $\$4$ per yard.

Here is a system of equations that represent the constraints in the situation.

$$\begin{cases} x+y=4.25 \\ 16.90x+4y=36.35 \end{cases}$$

1. What does the solution to the system represent?
2. Find the solution to the system of equations. Explain or show your reasoning.

Student Reflection: I think my math ability is _________________. Share why below.

a. Great and getting better   b. Decent and growing   c. A battle, but I am learning and growing

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What do your students think it means to be good at math? How are you helping them change negative impressions they might have about their ability to reason mathematically?

Practice Problems

1. The knitting club sold 40 scarves and hats at a winter festival and made $\$700$ from the sales. They charged $\$18$ for each scarf and $\$14$ for each hat.

   If s represents the number of scarves sold and h represents the number of hats sold, which system of equations represents the constraints in this situation?

   a. $\begin{cases} 40s + h = 700 \\ 18s + 14h = 700 \end{cases}$

   b. $\begin{cases} 18s + 14h = 40 \\ s + h = 700 \end{cases}$

   c. $\begin{cases} s + h = 40 \\ 18s + 14h = 700 \end{cases}$

   d. $\begin{cases} 40(s + h) = 700 \\ 18s = 14h \end{cases}$

2. Here are two equations:

   Equation 1: $6x + 4y = 34$
   Equation 2: $5x - 2y = 15$

   a. Decide whether each (x, y) pair is a solution to one equation, both equations, or neither of the equations.

      i. $(3, 4)$
      ii. $(4, 2.5)$
      iii. $(5, 5)$
      iv. $(3, 2)$

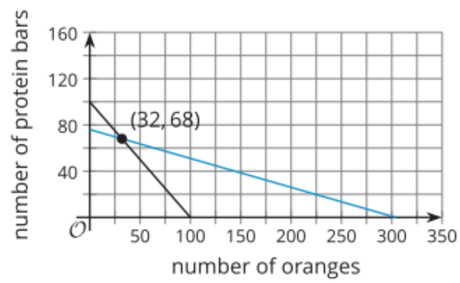
   b. Is it possible to have more than one (x, y) pair that is a solution to both equations? Explain or show your reasoning.

3. Explain or show that the point $(5, -4)$ is a solution to this system of equations: $\begin{cases} 3x - 2y = 23 \\ 2x + y = 6 \end{cases}$

4. A club is selling snacks at a track meet. Oranges cost $1 each and protein bars cost $4 each. They sell a total of 100 items, and collect $304.

   a. Write two equations that represent this situation.

   b. What does the solution $(32, 68)$ represent?



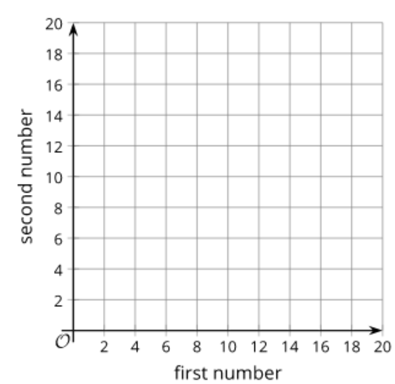
5. Diego is thinking of two positive numbers. He says, “If we triple the first number and double the second number, the sum is 34.”

   a. Write an equation that represents this clue. Then, find two possible pairs of numbers Diego could be thinking of.

   b. Diego then says, “If we take half of the first number and double the second, the sum is 14.”

      Write an equation that could represent this description.

   c. What are Diego’s two numbers? Explain or show how you know.
      A coordinate plane is given here, in case helpful.



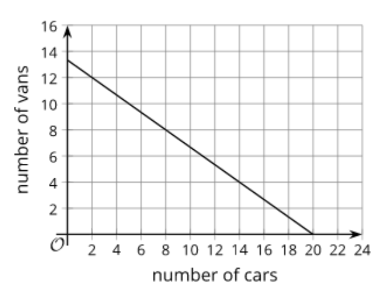
6. At a poster shop, Han paid $16.80 for two large posters and three small posters of his favorite band. Kiran paid $14.15 for one large poster and four small posters of his favorite TV shows. Posters of the same size have the same price.

   Find the price of a large poster, ℓ, and the price of a small poster, s.

7. Volunteer drivers are needed to bring 80 students to the championship baseball game. Drivers either have cars, which can seat four students, or vans, which can seat six students. The equation $4c + 6v = 80$ describes the relationship between the number of cars, c, and number of vans, v, that can transport exactly 80 students.

   Explain how you know that this graph represents this equation.

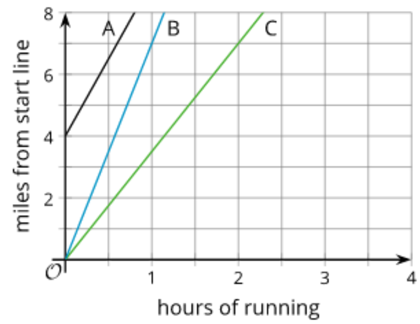
   (From Unit 3, Lesson 2)



8. Three siblings are participating in a family-friendly running event.

- The oldest sibling begins at the start line of the race and runs 7 miles per hour the entire time.
- The middle sibling begins at the start line and walks at 3.5 miles per hour throughout the race.
- The youngest sibling joins the race 4 miles from the start line and runs 5 miles per hour the rest of the way.

Match each graph to the sibling whose running is represented by the graph below.



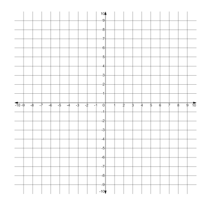
Graph	Sibling
Graph A Graph B Graph C	1. Oldest Sibling 2. Middle Sibling 3. Youngest Sibling

(From Unit 3, Lesson 6)

9. What is the x-intercept of the graph of $y = 3 - 5x$?

a. $(\frac{3}{5}, 0)$
b. $(-5, 0)$
c. $(0, 3)$
d. $(0, \frac{5}{3})$

(From Unit 3, Lesson 6)



10. A triangle has vertices $C(-3,-2)$, $A(2,5)$, and $T(6,-1)$. Is CAT isosceles? How do you know?

(From Unit 3, Lesson 7)

11. Match each equation with the corresponding equation solved for a.

a. $a + 2b = 5$
b. $5a = 2b$
c. $a + 5 = 2b$
d. $5(a + 2b) = 0$
e. $5a + 2b = 0$

1. $a = \frac{2b}{5}$
2. $a = \frac{-2b}{5}$
3. $a = -2b$
4. $a = 2b - 5$
5. $a = 5 - 2b$

(From Unit 2)

12. Andre does not understand why a solution to the equation $3 - x = 4$ must also be a solution to the equation $12 = 9 - 3x$

Write a convincing explanation as to why this is true.

(From Unit 2)

13. Priya's cell phone plan costs $35 each month plus $15 for each gigabyte of data she uses. Han's plan costs $75 each month, plus $5 for each gigabyte of data. Is there a number of gigabytes of data for which the plans cost the same amount? How many gigabytes will they use to pay the same amount?

(Addressing NC.8.EE.8)

Lesson 9: Solving Systems by Substitution

PREPARATION

Lesson Goals	Learning Targets
• Recognize that a system can be efficiently solved by substitution if one variable is already isolated or can be easily isolated. • Recognize that there are multiple ways to perform substitution to solve a system of equations. • Solve systems of linear equations by substituting a variable with a number or an expression, and check solutions by substituting them back into the equations.	• I can solve systems of equations by substituting a variable with a number or an expression. • I know more than one way to perform substitution and can decide which way or what to substitute based on how the given equations are written.

Lesson Narrative

In Lesson 8, students solved systems of linear equations by graphing. Here, they transition to solving systems algebraically.

In this lesson, students see that a system can be solved by replacing a variable with a number or with an expression, and that various substitutions can be done to solve the same system. They also begin to build an awareness of the kinds of systems that are conducive to being solved by substitution.

Students practice looking for and making use of structure as they identify the variables or expressions to substitute and ways to perform substitutions efficiently (MP7).

What do you hope to learn about your students during this lesson?

Focus and Coherence

Building On	Addressing
NC.8.EE.7: Solve real-world and mathematical problems by writing and solving equations and inequalities in one variable. • Recognize linear equations in one variable as having one solution, infinitely many solutions, or no solutions. • Solve linear equations and inequalities including multi-step equations and inequalities with the same variable on both sides **NC.M1.A-REI.10:** Understand that the graph of a two-variable equation represents the set of all solutions to the equation. **NC.M1.A-CED.1:** Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.	**NC.M1.A-REI.6:** Use tables, graphs, or algebraic methods (substitution and elimination) to find approximate or exact solutions to systems of linear equations and interpret solutions in terms of a context.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U3.L9 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.8.EE.7; NC.M1.A-CED.1

The purpose of this bridge is to support students with substituting a value for a variable into an expression then evaluating the result. These skills help students develop fluency and will be helpful later in this lesson when students will need to be able to substitute for a variable in equations.

Student Task Statement

Find the value of y when $x = 5$.

a. $y = 3x - 4$
b. $y = \frac{2}{5}x + 4$
c. $y = 2x + 3 + (3x - 1)$
d. $y = 4x - (x + 1)$

DO THE MATH	PLANNING NOTES

Warm-up: Is It a Match? (*5 minutes*)

Instructional Routines: Math Talk; Discussion Supports (MLR8) - Responsive Strategy

Building On: NC.M1.A-REI.10

This *Math Talk* encourages students to look for connections between the features of graphs and of linear equations that each represent a system. Given two graphs on an unlabeled coordinate plane, students must rely on what they know about horizontal and vertical lines, intercepts, and slope to determine if the graphs could represent each pair of equations. The equations presented and the reasoning elicited here will be helpful later in the lesson, when students solve systems of equations by substitution.

All four systems include an equation for either a horizontal or a vertical line. Some students may remember that the equation for such lines can be written as $x = a$ or $y = b$, where a and b are constants. (In each of the first three systems, one equation is already in this form. In the last system, a simple rearrangement to one equation would put it in this form.) Activating this knowledge would enable students to quickly tell whether a system matches the given graphs.

Those who don't recall it can still reason about the system structurally. For instance, given a system with $x =- 5$ as one of the equations, they may reason that any point that has a negative x-value will be to the left of the vertical axis. The solution (if there is one) to this system would have to have -5 for the x-value. The intersection of the given graphs is a point to the right of the vertical axis (and therefore having a positive x-value), so the graphs cannot represent that system.

To match graphs and equations, students need to look for and make use of structure (MP7) in both representations. In explaining their strategies, students need to be precise in their word choice and use of language (MP6).

Because the warm-up is intended to promote reasoning, discourage the use of graphing technology to graph the systems.

RESPONSIVE STRATEGY

To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

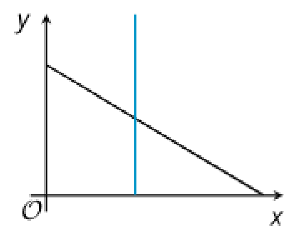
Step 1

- Display the graphs and systems and ask students to choose at least two systems to explore.
- Give students 1–2 minutes of quiet think time and ask them to give a signal when they have their answers and are ready to explain their strategies.

Student Task Statement

Here are graphs of two equations in a system.

Choose two of these systems and determine if each of them could be represented by the graphs. Be prepared to explain how you know.



a. $\begin{cases} x + 2y = 8 \\ x = -5 \end{cases}$ b. $\begin{cases} y = -7x + 13 \\ y = -1 \end{cases}$ c. $\begin{cases} 3x = 8 \\ 3x + y = 15 \end{cases}$ d. $\begin{cases} y = 2x - 7 \\ 4 + y = 12 \end{cases}$

Step 2

- Facilitate a whole-class discussion by asking students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:
  - "Who can restate ___'s reasoning in a different way?"
  - "Did anyone have the same strategy but would explain it differently?"
  - "Did anyone solve the problem in a different way?"
  - "Does anyone want to add on to ___'s strategy?"
  - "Do you agree or disagree? Why?"
- If any systems were not chosen by students, skip them at first, then ask students what ideas they have about those systems now that they've heard their classmates' reasoning.

RESPONSIVE STRATEGIES

To support students to explain their strategy, display a word wall and a sentence frame to support students to explain their strategy. Include the following on the word wall: intersect, intercept, vertical, solution, values. Display this sentence frame: "I noticed __, so I __." Invite students to copy the graph by hand in their notes, using black and blue ink. Some students may benefit from the opportunity to rehearse with a partner before they share with the whole class.

Discussion Supports (MLR8)

- If no students mentioned solving the systems and then checking to see if the solution could match the graphs, ask if anyone approached it that way. For instance, ask: “How could we find the solution to system “b” without graphing?” Give students a moment to discuss their ideas with a partner and then proceed to the next activity.

DO THE MATH	PLANNING NOTES

Activity 1: Four Systems (*15 minutes*)

Instructional Routine: Compare and Connect (MLR7)
Addressing: NC.M1.A-REI.6

In this activity, students see the same four pairs of equations as those in the warm-up. This time, their job is to find a way to solve the systems. Some students may choose to solve by graphing, but the systems lend themselves to be solved efficiently and precisely by substitution.

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Provide students 6 minutes of quiet time to solve as many systems as they can and then a couple of minutes to share their responses and strategies with their partner.
- Ask students to prepare a visual display of their work to solve one of the systems. Encourage at least two pairs of students to create a visual display of their work on system “d.”

RESPONSIVE STRATEGIES

Demonstrate use of highlighter or color to keep track of x and y during the work process. Emphasize the importance of the equals sign with each step.

Chunk this task in half to differentiate the degree of difficulty or complexity. Present and discuss the first two problems before showing the last two. Support students in metacognitive development by having students articulate the steps they used to solve by substitution in these more straightforward examples. Record the steps on a chart or whiteboard to encode the process for later retrieval. Then, reveal the remaining problems.

Supports accessibility for: Organization; Attention

Monitoring Tip: As students work, pay attention to the methods students use to solve the systems. Identify those who solve by graphing and those who solve by substitution—by replacing a variable or an expression in one equation with an equal value or equivalent expression from the other equation. Ask these students to be prepared to share later.

Advancing Student Thinking: Some students may not remember to find the value of the second variable after finding the first. They may need a reminder that the solution to a system of linear equations is a pair of values.

If some students struggle with the last system because the variable that is already isolated is equal to an expression rather than a number, ask what they would do if the first equation were $y = a\ number$ instead of $y = 2x - 7$.

If students don't know how to approach the last system, ask them to analyze both equations and see if the value of one of the variables could be found easily.

Student Task Statement

Here are four systems of equations you saw earlier. Solve each system. Then, check your solutions by substituting them into the original equations to see if the equations are true.

a. $\begin{cases} x+2y=8 \\ x=-5 \end{cases}$

b. $\begin{cases} y=-7x+13 \\ y=-1 \end{cases}$

c. $\begin{cases} 3x=8 \\ 3x+y=15 \end{cases}$

d. $\begin{cases} y=2x-7 \\ 4+y=12 \end{cases}$

Step 2

- Use the *Compare and Connect* routine to facilitate a whole-class discussion by selecting previously identified students to share their responses and strategies. Display their work for all to see. Highlight one strategy that involves graphing and at least one strategy that involves substitution, and amplify the variety of language students use to explain their process for substitution.
- Make sure students see that the last two equations can be solved by substituting in different ways.
  - Here are two ways for solving the third system, $\begin{cases} 3x=8 \\ 3x+y=15 \end{cases}$, by substitution:

Finding the value of x and substituting it into $3x+8=15$:

$$3x=8$$
$$x=\frac{8}{3}$$

$$3x+y=15$$
$$3(\tfrac{8}{3})+y=15$$
$$8+y=15$$
$$y=7$$

Substituting the value of $3x$ into $3x+8=15$

$$3x+y=15$$
$$8+y=15$$
$$y=7$$

  - Here are two ways of solving the last system, $\begin{cases} y=2x-7 \\ 4+y=12 \end{cases}$, by substitution:

Substituting $2x-7$ for y in the equation $4+y=12$:

Rearranging or solving $4+y=12$ to get $y=8$, and then substituting 8 for y in $2x-7$:

$$\begin{aligned} 4+y &= 12 \\ 4+(2x-7) &= 12 \\ 4+2x-7 &= 12 \\ 2x-7+4 &= 12 \\ 2x-3 &= 12 \\ 2x &= 15 \\ x &= 7.5 \end{aligned}$$

$$\begin{aligned} y &= 2x-7 \\ y &= 2(7.5)-7 \\ y &= 15-7 \\ y &= 8 \end{aligned}$$

$$\begin{aligned} y &= 2x-7 \\ 8 &= 2x-7 \\ 15 &= 2x \\ 7.5 &= x \end{aligned}$$

- In each of these two systems, students are likely to notice that one way of substituting is much quicker than the other. Emphasize that when one of the variables is already isolated or can be easily isolated, substituting the value of that variable (or the expression that is equal to that variable) into the other equation in the system can be an efficient way to solve the system.

DO THE MATH	PLANNING NOTES

Activity 2: What about Now? (*10 minutes*)

Instructional Routine: Discussion Supports (MLR8) - Responsive Strategy	
Addressing: NC.M1.A-REI.6	

The activity allows students to practice solving systems of linear equations by substitution and reinforces the idea that there are multiple ways to perform substitution. Students are directed to find the solutions without graphing.

Monitor for the different ways that students use substitutions to solve the systems. Invite students with different approaches to share later.

Step 1

- Keep students in pairs.
- Provide students a few minutes to work quietly and then time to discuss their work with a partner. If time is limited, ask each partner to choose two different systems to solve.

RESPONSIVE STRATEGIES

Differentiate the degree of difficulty or complexity by allowing students to complete problems in any order. Students may benefit from solving the system that uses the variables x and y first. Highlight connections between representations by inviting students use the same color to highlight a variable and the expression to be substituted for that variable.

Supports accessibility for: Conceptual processing

***Advancing Student Thinking*:** When solving the second system, students are likely to substitute the expression $2m+10$ for p in the first equation, $2m-2p=-6$. Done correctly, it should be written as $2m-2(2m+10)=-6$. Some students may neglect to write parentheses and write $2m-4m+10=-6$. Remind students that if p is equal to $2m+10$, then $2p$ is 2 times $2m+10$ or $2(2m+10)$. (Alternatively, use an example with a sum of two numbers for p: Suppose $p=10$, which means $2p=2(10)$, or 20. If we express p as a sum of 3 and 7, or $p=3+7$, then $2p=2(3+7)$, not $2\cdot 3+7$. The latter has a value of 13, not 20.)

Some students who correctly write $2m-2(2m+10)=-6$ may fail to distribute the subtraction and write the left side as $2m-4m+20$. Remind them that subtracting $2(2m+10)$ can be thought of as adding $-2(2m+10)$ and ask how they would expand this expression.

Student Task Statement

Solve each system without graphing.

a. $\begin{cases} 5x-2y=26 \\ y+4=x \end{cases}$

b. $\begin{cases} 2m-2p=-6 \\ p=2m+10 \end{cases}$

c. $\begin{cases} 2d=8f \\ 18-4f=2d \end{cases}$

d. $\begin{cases} w+\frac{1}{7}z=4 \\ z=3w-2 \end{cases}$

Are You Ready For More?

Solve this system with four equations.

$$\begin{cases} 3x+2y-z+5w=20 \\ y=2z-3w \\ z=w+1 \\ 2w=8 \end{cases}$$

Step 2

- Facilitate a whole-class discussion by selecting previously identified students to share their responses and reasoning. Display their work for all to see.
- Highlight the different ways to perform substitutions to solve the same system. For example:
  - In the second system, $\begin{cases} 2m-2p=-6 \\ p=2m+10 \end{cases}$, we could substitute $2m+10$ for p in the first equation, or we could substitute $p-10$ for $2m$ in the first equation.
  - In the third system, $\begin{cases} 2d=8f \\ 18-4f=2d \end{cases}$, we could substitute $8f$ for $2d$ in the second equation, or we could substitute $\frac{1}{4}d$ for f in the second equation.

RESPONSIVE STRATEGY

Model explicit use of sample sentences by use of frames, or once responses like the samples are generated, write on board/capture on chart paper the complete statement(s). For example, "In the second system, we could substitute ____ for ____ in the first equation, or we could substitute ____ for ____ in the first equation.

Discussion Supports (MLR8)

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

To review techniques for solving by substitution, facilitate a class discussion on the following questions. The first two questions focus on strategies for isolating a variable, while the following three focus on mechanics.

Present the following systems to students one at a time:

System A:

$$\begin{cases} 3x - 2y = -4 \\ 2x = 6y \end{cases}$$

System B:

$$\begin{cases} x - 2y = 6 \\ 5x - 2y = -10 \end{cases}$$

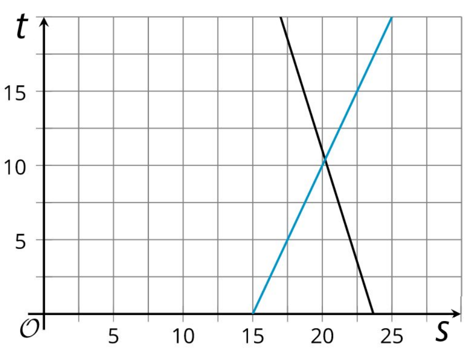
- "When solving system A, what substitution would you make?" (Isolate x in the second equation because the resulting equation is easiest to work with. Students may also make valid arguments for isolating y, including that they would rather work with the variable x after the substitution)
- "When solving system B, what substitution would you make?" (Isolate x in the first equation because you can do it in one step. Isolate $2y$ in either the first or second equation because it can be substituted directly in for $2y$ in the second equation.)
- "If we find $2y = x - 6$ and substitute $x - 6$ for $2y$ in the second equation, what equation results?" $(5x - 2(x - 6) = -10)$
- "How can we simplify the left side?" (Distribute the -2 and combine like terms to get $3x + 12 = -10$)
- "Is $(-1, 3)$ a solution to system B? How can we check?" (No, because when you finish solving you find that $(-1, -2.5)$ is the solution; No, because when you substitute $(-1, 3)$ into either of the two equations, the equations are false.)

PLANNING NOTES

Student Lesson Summary and Glossary

The solution to a system can usually be found by graphing, but graphing may not always be the most precise or the most efficient way to solve a system.

Here is a system of equations: $\begin{cases} 3s+t=71 \\ 2s-t=30 \end{cases}$



The graphs of the equations show an intersection point at approximately 20 for s and approximately 10 for t.

Without technology, however, it is not easy to tell what the exact values are.

Instead of solving by graphing, we can solve the system algebraically. Here is one way.

If we subtract $3s$ from each side of the first equation, $3s+t=71$, we get an equivalent equation: $t=71-3s$. Rewriting the original equation this way allows us to isolate the variable t.

Because t is equal to $71-3s$, we can substitute the expression $71-3s$ in the place of t in the second equation. Doing this gives us an equation with only one variable, s, and makes it possible to find s.

$$\begin{aligned} 2s-t&=30 && \text{original equation} \\ 2s-(71-3s)&=30 && \text{substitute } 71-3s \text{ for } t \\ 2s-71+3s&=30 && \text{apply distributive property} \\ 5s-71&=30 && \text{combine like terms} \\ 5s&=101 && \text{add 71 to both sides} \\ s&=\tfrac{101}{5} && \text{divide both sides by 5} \\ s&=20.2 \end{aligned}$$

Now that we know the value of s, we can find the value of t by substituting 20.2 for s in either of the original equations and solving the equation.

$$\begin{aligned} 3(20.02)+t&=71 \\ 60.6+t&=71 \\ t&=71-60.6 \\ t&=10.4 \end{aligned}$$

$$\begin{aligned} 2(20.2)-t&=30 \\ 40.4-t&=30 \\ -t&=30-40.4 \\ -t&=-10.4 \\ t&=\tfrac{-10.4}{-1} \\ t&=10.4 \end{aligned}$$

The solution to the system is the pair $s=20.2$ and $t=10.4$, or the point $(20.2, 10.4)$ on the graph.

This method of solving a system of equations is called solving by *substitution*, because we substituted an expression for t into the second equation.

Cool-down: A System to Solve (*5 minutes*)

Addressing: NC.M1.A-REI.6

Cool-down Guidance: Points to Emphasize
In a subsequent lesson, highlight both ways to substitute and invite students to discuss the benefits or potential challenges in each. (The lesson synthesis for Lesson 10 is a place where substitution may be discussed again.)

Cool-down

Solve this system of equations without graphing and show your reasoning:

$$\begin{cases} 5x + y = 7 \\ 20x + 2 = y \end{cases}$$

Student Reflection:

The teaching strategies being used to help me learn are:

a. Great! b. Good, but I'd like something different c. Not helping me at all

Feel free to give details on what teaching strategies are helping most and which you'd change.

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Reflect on how comfortable your students are asking questions of you and of each other. What can you do to encourage students to ask questions?

Practice Problems

1. Identify a solution to this system of equations: $\begin{cases} -4x+3y=23 \\ x-y=-7 \end{cases}$

   a. $(-5,2)$

   b. $(-2,5)$

   c. $(-3,4)$

   d. $(4,-3)$

2. Lin is solving this system of equations: $\begin{cases} 6x-5y=34 \\ 3x+2y=8 \end{cases}$

   She starts by rearranging the second equation to isolate the y variable: $y=4-1.5x$. She then substituted the expression $4-1.5x$ for y in the first equation, as shown:

   $$\begin{aligned} 6x-5(4-1.5x)&=34 \\ 6x-20-7.5x&=34 \\ -1.5x&=54 \\ x&=-36 \end{aligned}$$

   $$\begin{aligned} y&=4-1.5x \\ y&=4-1.5\cdot(-36) \\ y&=58 \end{aligned}$$

   a. Check to see if Lin's solution of $(-36,58)$ makes both equations in the system true.

   b. If your answer to the previous question is "no," find and explain her mistake. If your answer is "yes," graph the equations to verify the solution of the system.

3. Solve each system of equations.

   a. $\begin{cases} 2x-4y=20 \\ x=4 \end{cases}$

   b. $\begin{cases} y=6x+11 \\ 2x-3y=7 \end{cases}$

4. Tyler and Han are trying to solve this system by substitution: $\begin{cases} x+3y=-5 \\ 9x+3y=3 \end{cases}$

   Tyler's first step is to isolate x in the first equation to get $x=-5-3y$. Han's first step is to isolate $3y$ in the first equation to get $3y=-5-x$.

   Show that both first steps can be used to solve the system and will yield the same solution.

5. Elena wanted to find the slope and y-intercept of the graph of $25x - 20y = 100$. She decided to put the equation in slope-intercept form first. Here is her work:

$$25x - 20y = 100$$
$$20y = 100 - 25x$$
$$y = 5 - \frac{5}{4}x$$

She concluded that the slope is $-\frac{5}{4}$ and the y-intercept is $(0, 5)$.

   a. What was Elena's mistake?

   b. What are the slope and y-intercept of the line? Explain or show your reasoning.

(From Unit 3, Lesson 3)

6. Find the x- and y-intercepts of the graph of each equation.

   a. $y = 10 - 2x$

   b. $4y + 9x = 18$

   c. $6x - 2y = 44$

   d. $2x = 4 + 12y$

(From Unit 3, Lesson 3)

7. Andre is buying snacks for the track and field team. He buys a pounds of apricots for $\$6$ per pound and b pounds of dried bananas for $\$4$ per pound. He buys a total of 5 pounds of apricots and dried bananas, and spends a total of $\$24.50$.

Which system of equations represents the constraints in this situation?

   a. $\begin{cases} 6a + 4b = 5 \\ a + b = 24.50 \end{cases}$

   b. $\begin{cases} 6a + 4b = 24.50 \\ a + b = 5 \end{cases}$

   c. $\begin{cases} 6a = 4b \\ 5(a + b) = 24.50 \end{cases}$

   d. $\begin{cases} 6a + b = 4 \\ 5a + b = 24.50 \end{cases}$

(From Unit 3, Lesson 8)

8. Here are two equations:

Equation 1: $y = 3x + 8$
Equation 2: $2x - y = -6$

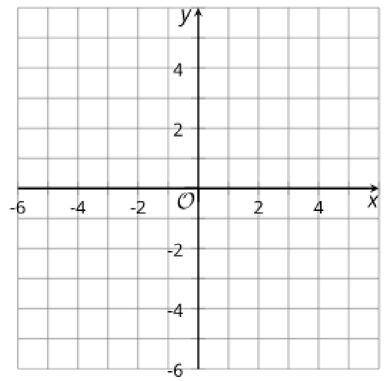
Without using graphing technology:

a. Find a point that is a solution to equation 1 but not a solution to equation 2.

b. Find a point that is a solution to equation 2 but not a solution to equation 1.

c. Graph the two equations.

d. Find a point that is a solution to both equations.



(From Unit 3, Lesson 8)

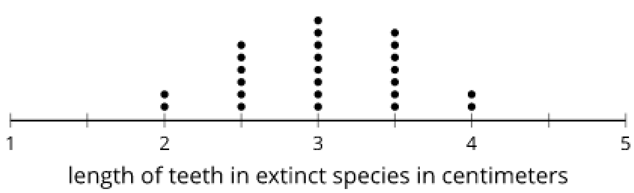
9. Kiran buys supplies for the school's greenhouse. He buys f bags of fertilizer and p packages of soil. He pays $\$5$ for each bag of fertilizer and $\$2$ for each package of soil, and spends a total of $\$90$. The equation $5f + 2p = 90$ describes this relationship.

If Kiran solves the equation for p, which equation would result?

a. $2p = 90 - 5f$
b. $p = \frac{5f-90}{2}$
c. $p = 45 - 2.5f$
d. $p = \frac{85f}{2}$

(From Unit 2)

10. The dot plots show the distribution of the length, in centimeters, of 25 shark teeth for an extinct species of shark and the length, in centimeters, of 25 shark teeth for a closely related shark species that is still living.



mean: 3.02 cm
standard deviation: 0.55 cm

mean: 2.32 cm
standard deviation: 0.13 cm

Compare the two dot plots using the shape of the distribution, measures of center, and measures of variability. Use the situation described in the problem in your explanation.

(From Unit 1)

11. For each equation, find the value of y when $x = -3$.

a. $y = 4x$

b. $y = -4x + 2$

c. $y = -(4x + 2)$

d. $y = 4x - 2$

(Addressing NC.8.EE.7)

Lesson 10: Solving Systems by Elimination (Part One)

PREPARATION

Lesson Goal	Learning Target
• Solve systems of equations by adding or subtracting the equations strategically to eliminate a variable.	• I can solve systems of equations by adding or subtracting them to eliminate a variable.

Lesson Narrative

This is the first of two lessons that develop the idea of solving systems of linear equations in two variables by elimination. The first lesson introduces adding or subtracting the equations in the system, resulting in a one-variable equation that can then be solved. The second lesson addresses the need to multiply one or both of the equations in a system to make it possible to eliminate a variable.

Students warm up to the idea of adding equations visually. They examine a diagram of three hangers where the third hanger contains the combined contents of the first two hangers, and all three hangers are balanced. Then, they analyze the result of adding two linear equations in standard form and notice that doing so eliminates one of the variables, enabling them to solve for the other variable and, consequently, to solve the system. In studying and testing a new strategy of adding equations and then offering their analyses, students construct viable arguments and critique the reasoning of others (MP3).

Students also practice solving systems by adding and subtracting equations and checking their solutions. They also encounter systems where one variable cannot be easily eliminated (given what they know at this point), motivating the need for another strategy.

Which Standards for Mathematical Practice do you anticipate students engaging in during this lesson? How will you support them?

Focus and Coherence

Building On	Addressing
NC.6.EE.4: Identify when two expressions are equivalent and justify with mathematical reasoning.	**NC.M1.A-REI.5:** Explain why replacing one equation in a system of linear equations by the sum of that equation and a multiple of the other produces a system with the same solutions. **NC.M1.A-REI.6:** Use tables, graphs, or algebraic methods (substitution and elimination) to find approximate or exact solutions to systems of linear equations and interpret solutions in terms of a context.

Agenda, Materials, and Preparations

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
  - Graphing technology is optional for this lesson as students may use it to check solutions in Activity 2. Acquire devices that can access Desmos (recommended) or other graphing technology. It is ideal if each student has their own device.
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U3.L10 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.6.EE.4

The purpose of this bridge is for students to practice combining like terms. Students will need to add or subtract similar expressions to eliminate a variable when solving systems of equations by elimination. This activity gives students the chance to practice that skill.

Student Task Statement

Rewrite each expression by combining like terms.

1. $11s - 2s$
2. $-4x + 6y - (7x + 2y)$
3. $8x - 3y + (3y - 5x)$
4. $5x + 4y - (5x + 7y)$

DO THE MATH	PLANNING NOTES

Warm-up: Hanger Diagrams (*5 minutes*)

Instructional Routine: Notice and Wonder	
Building Towards: NC.M1.A-REI.5	

The purpose of this *Notice and Wonder* warm-up is to give students an intuitive and concrete way to think about combining two equations that are each true.

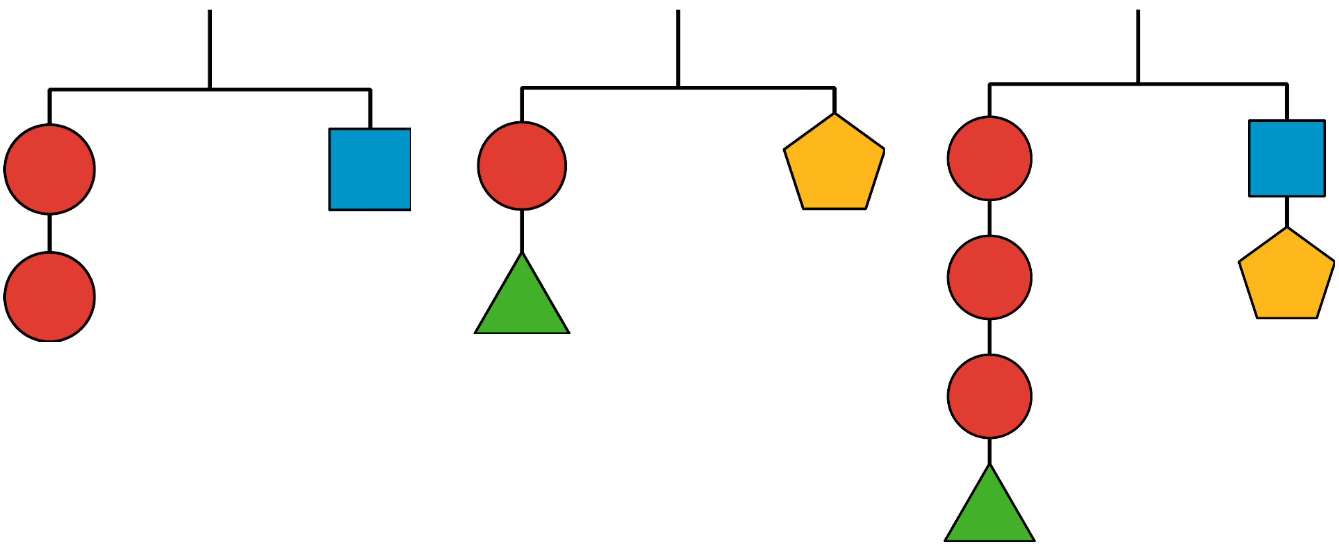
Students are presented with diagrams of three balanced hangers, which suggest that the weights on the two sides of each hanger are equal. Each side of the last hanger shows the combined objects from the corresponding side of the first two hangers. Students can reason that if two circles weigh the same as one square, and one circle and one triangle weigh the same as one pentagon, then the combined weight of three circles and one triangle should also be equal to the combined weight of one square and one pentagon.

Step 1

- Display the hanger diagrams for all to see.
- Ask students to think of at least one thing they notice and at least one thing they wonder.
- Give students 1 minute of quiet think time
- Give students 1 minute to discuss the things they notice and wonder with a partner.

Student Task Statement

What do you notice? What do you wonder?



Step 2

- Facilitate a whole-class discussion by asking students to share the things they noticed and wondered.
- Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image.
- After all responses have been recorded without commentary or editing, ask students, “Is there anything on this list that you are wondering about now?” Encourage students to respectfully disagree, ask for clarification, or point out contradicting information.

- The idea to emphasize is that the weights on each side of the third hanger come from combining the weights on the corresponding sides of the first two hangers. If no one points this out, raise it as a point for discussion. Ask students:
  - “What do you notice about the left side of the last hanger? What about the right side?"
  - "If we only saw the first two hangers but knew that the third hanger has the combined weights from the corresponding side of the first two hangers, could we predict whether the weights on the third hanger would balance? Why or why not?” (Yes. We can think of it as adding the weights from the second hanger to the first one, or vice versa. If the same weight is added to each side of a balanced hanger, the hanger would still be balanced.)

DO THE MATH	PLANNING NOTES

Activity 1: Adding Equations (*15 minutes*)

Instructional Routine: Stronger and Clearer Each Time (MLR1)
Addressing: NC.M1.A-REI.5; NC.M1.A-REI.6

In the warm-up of this lesson, students saw a visual representation of two equations being added to form a third equation. Because the first two equations are balanced, the third is also balanced. In this activity, students continue to develop the idea of adding two equations to form a third equation and use it to help them solve systems of linear equations.

Along the way, students examine the work of others and practice explaining their own reasoning as well as critiquing that of others (MP3). They also see that sometimes adding equations is a productive way to solve systems, but other times it isn't.

RESPONSIVE STRATEGIES

Explicitly link the warm-up to the representation involved in analyzing Diego’s work. Connect the intuitive understanding of balance to familiar mathematical statements that do not require solving. Present strategically, in the same format as Diego’s work. For instance, ask students if 2+2=4 is a true and balanced statement. Then, if 3+1=4 is also a true and balanced statement. Once students have labeled these as true, add them together in the same arrangement as the upcoming problems and ask students if the result (5+3=8), will therefore also be true and balanced. Then, introduce Diego’s work.

Supports accessibility for: Conceptual processing; Memory

Step 1

- Ask students to arrange themselves into pairs or use visibly random grouping.
- Provide students 2 minutes of quiet time to think about the first set of questions and then time to share their thinking with their partner.
- Follow with a whole-class discussion before students proceed to the second set of questions. Invite students to share their analysis of Diego's work—what Diego has done to solve the system and why he might have done it that way. Discuss questions such as:
  - "In this case, what happens when the equations are added? Why might it be helpful to do so?" (The expressions with x add up to 0, so it's removed from the equation, making it possible to solve for y.)

- "How does finding the value of y help with solving the system?" (Once we know the value of one variable, we can use it to find the value of the other, by substituting it back into one of the equations and solving that equation.)
- "How can we be sure that $x = 1$ and $y = 2$ make both equations true and that $(1, 2)$ is a solution to the system?" (We can substitute those values into the equations and see if the resulting equations are true. We can also graph the system and see if it intersects at $(1, 2)$.)

- Next, ask students to complete the remainder of the activity.

Student Task Statement

Diego is solving this system of equations:

$$\begin{cases} 4x + 3y = 10 \\ -4x + 5y = 6 \end{cases}$$

Here is his work:

$$\begin{array}{rr} & 4x + 3y = 10 \\ + & (-4x + 5y = 6) \\ \hline & 0 + 8y = 16 \\ & y = 2 \end{array}$$

$$\begin{aligned} 4x + 3(2) &= 10 \\ 4x + 6 &= 10 \\ 4x &= 4 \\ x &= 1 \end{aligned}$$

1. Make sense of Diego's work and discuss with a partner:
   a. What did Diego do to solve the system?
   b. Is the pair of x and y values that Diego found actually a solution to the system? How do you know?
2. Does Diego's method work for solving these systems? Be prepared to explain or show your reasoning.
   a. $\begin{cases} 2x + y = 4 \\ x - y = 11 \end{cases}$
   b. $\begin{cases} 8x + 11y = 37 \\ 8x + y = 7 \end{cases}$

Step 2

- Invite students to share their responses to the last set of questions and discuss whether Diego's method works for solving the two systems. Possible questions to ask students:
  - "Why does adding the equations work for solving the system with $2x + y = 4$ and $x – y = 11$, but not for $8x + 11y = 37$ and $8x + y = 7$?" (When the equations in the first system are added, the y variable is eliminated. When the equations in the second system are added, there are still two variables whose values we don't know.)
  - "What if we subtract the equations? Would that help us solve the last system?" (Yes, subtracting the second equation from the first gives $10y = 30$ or $y = 3$, which we can then use to find the x-value.)

Use the *Stronger and Clearer Each Time* routine to help students connect the hanger diagram in the warm-up with adding equations in this activity. Give students 1 minute to write a first draft response to the question, "How is adding the two equations here like and unlike combining the shapes in the two hanger diagrams earlier?" Provide listeners with prompts for feedback that will help their partner add details to strengthen and clarify their ideas. For example: "Can you give an example…. ?" or "Can you say more about....?" Give students 2–3 minutes to revise their initial draft based on feedback from their peers.

- Call attention to any student expression of the idea that if we choose to add or subtract strategically, one variable is eliminated, making it possible to solve for the other variable. When the value of that variable is substituted into either of the original equations, we can solve for the variable that was eliminated. Tell students that this method of solving a system is called solving by elimination.
- Point out that there is nothing wrong about adding the equations in the last system. It simply doesn't get us anywhere closer to the solution and is therefore unproductive.

DO THE MATH	PLANNING NOTES

Activity 2: A Bunch of Systems (*10 minutes*)

Instructional Routine: Critique, Correct, Clarify (MLR3) - Responsive Strategy
Addressing: NC.M1.A-REI.6

In this activity, students practice using algebra to solve systems of linear equations in two variables and checking their solutions. Students do not have to use elimination, but the equations in the first two systems conveniently have opposites for the coefficients of one variable, so one variable can be easily eliminated. For the third system, students can subtract the two equations to eliminate a variable.

In the last system, none of the coefficients of the variables are opposites. Some students may choose to solve the system by substitution, but the process would be pretty cumbersome. The complication students encounter here motivates the need for another move, which students will explore in the next lesson.The activity concludes with a full class discussion and an option for engaging students in the *Critique, Correct, Clarify* routine.

Students who opt to use technology to check their solutions practice choosing tools strategically (MP5).

CRITIQUE, CORRECT, CLARIFY	***What Is This Routine?*** Students are given a piece of mathematical writing that is not their own to analyze, reflect on, and develop. The writing is an incorrect, incomplete, or unclear "first draft" written argument or explanation, and students' job is to improve the writing by correcting any errors, clarifying meaning, and adding explanation, justification, or details. The routine begins with a brief critique of the first draft in which the teacher elicits two or three ideas from students to identify what could use improvement; students then individually write second drafts, and finally, the teacher scribes as two or three students read their second drafts aloud. ***Why This Routine?*** *Critique, Correct, Clarify* (MLR3) prompts student reflection, fortifies output, and builds students' meta-linguistic awareness. The final step of public scribing creates an opportunity to invite all students to help edit a final draft together, so that it makes sense to more people. Teachers can demonstrate with meta-think-alouds and press for details when necessary.

Step 1

- Keep students in pairs.
- Provide access to graphing technology in case it is needed for checking solutions.
- If time is limited, ask one partner in each group to solve the first two of the systems and the other partner to solve the last two, and then check each other's solutions.

Student Task Statement

Solve each system of equations without graphing and show your reasoning. Then, check your solutions.

1. $\begin{cases} 2x+3y=7 \\ -2x+4y=14 \end{cases}$
2. $\begin{cases} 2x+3y=7 \\ 3x-3y=3 \end{cases}$
3. $\begin{cases} 2x+3y=5 \\ 2x+4y=9 \end{cases}$
4. $\begin{cases} 2x+3y=16 \\ 6x-5y=20 \end{cases}$

Are You Ready For More?

This system has three equations: $\begin{cases} 3x+2y-z=7 \\ -3x+y+2z=-14 \\ 3x+y-z=10 \end{cases}$

1. Add the first two equations to get a new equation.
2. Add the second two equations to get a new equation.
3. Solve the system of your two new equations.
4. What is the solution to the original system of equations?

Step 2

- Invite students to share their solutions and strategies for the first three systems and how they checked their solutions. Then, focus the discussion on the last system. Solicit the strategies students used for approaching that system. If someone solved it by substitution, display the work for all to see. If no one did, ask if it is possible to do. (If time permits, consider asking students to attempt to do so, or demonstrate that strategy to illustrate that it is not exactly efficient.)
- Discuss questions such as:
  - "What happens if we add or subtract the equations?" (No variables are eliminated, so it doesn't help with solving.)
  - "Why were you able to solve the first three systems by elimination but not the last one?" "What features were in the equations in those systems but not in the last one?" (In the first three systems, at least one variable in each pair of equations had the same or opposite coefficients, so when the terms were added or subtracted, the result was 0.)
- We need new moves! Tell students that they will explore another way to solve a system by elimination in an upcoming lesson.

RESPONSIVE STRATEGY

Use the Critique, Correct, Clarify routine by providing students with the following first draft writing to improve: "Solving a system of equations means finding x and y to make it true for both equations. Adding and subtracting the equations can get rid of x or y, then it's easier to find a solution to plug in to the other one." Ask students to first identify any parts of the writing that could be more clear or complete, and have them share 2-3 ideas aloud, annotating the first draft as they share. Then ask students to write a second draft that is more clear and more complete than the first draft. Invite students to share and compare their second drafts, and finally scribe as 1-2 students read their second draft aloud.

Critique, Correct, Clarify (MLR3)

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

Now that students have an additional strategy for solving systems in their toolkit, emphasize that the method we choose for solving a system may depend on the structure of that system. Some systems are more conducive to being solved by substitution while others are suited better for elimination.

Display all three systems.

System 1:

$$\begin{cases} 3m + n = 71 \\ 2m - n = 30 \end{cases}$$

System 2:

$$\begin{cases} 4x + y = 1 \\ y = -2x + 9 \end{cases}$$

System 3:

$$\begin{cases} 5x + 4y = 15 \\ 5x + 11y = 22 \end{cases}$$

- Ask students to decide if they would use substitution or elimination to solve the system and be prepared to explain why. It is not expected that they solve the system.
- Provide students 1 minute of quiet think time to decide and then time to discuss with a partner.
- Ask students to share their decision and explain how the structure of the system influenced their solution method. (when the coefficients are the same or opposite such as in systems 1 and 3, then subtracting or adding the equations will eliminate one of the variables; when one of the variables is isolated such as in system 2, then use substitution)

PLANNING NOTES

Student Lesson Summary and Glossary

Another way to solve systems of equations algebraically is by *elimination*. Just like in substitution, the idea is to eliminate one variable so that we can solve for the other. This can be done by adding or subtracting equations in the system. Let’s look at an example.

$$\begin{cases} 5x + 7y = 64 \\ 0.5x - 7y = -9 \end{cases}$$

Notice that one equation has $7y$ and the other has $-7y$.

If we add the second equation to the first, the $7y$ and $-7y$ add up to 0, which eliminates the y-variable, allowing us to solve for x.

$$\begin{array}{rr} & 5x + 7y = 64 \\ + & (0.5x - 7y = -9) \\ \hline & 5.5x + 0 = 55 \\ & 5.5x = 55 \\ & x = 10 \end{array}$$

Now that we know $x = 10$, we can substitute 10 for x in either of the equations and find y:

$$\begin{aligned} 5x + 7y &= 64 \\ 5(10) + 7y &= 64 \\ 50 + 7y &= 64 \\ 7y &= 14 \\ y &= 2 \end{aligned} \qquad \begin{aligned} 0.5x - 7y &= -9 \\ 0.5(10) - 7y &= -9 \\ 5 - 7y &= -9 \\ -7y &= -14 \\ y &= 2 \end{aligned}$$

In this system, the coefficient of y in the first equation happens to be the opposite of the coefficient of y in the second equation. The sum of the terms with y-variables is 0.

What if the equations don't have opposite coefficients for the same variable, like in the following system?

$$\begin{cases} 8r + 4s = 12 \\ 8r + s = -3 \end{cases}$$

Notice that both equations have $8r$ and if we subtract the second equation from the first, the variable r will be eliminated because $8r - 8r$ is 0.

$$\begin{array}{rr} & 8r + 4s = 12 \\ - & (8r + s = -3) \\ \hline & 0 + 3s = 15 \\ & 3s = 15 \\ & s = 5 \end{array}$$

Substituting 5 for s in one of the equations gives us r:

$$\begin{aligned} 8r + 4s &= 12 \\ 8r + 4(5) &= 12 \\ 8r + 20 &= 12 \\ 8r &= -8 \\ r &= -1 \end{aligned}$$

Cool-down: What to Do with This System? (*5 minutes*)

Addressing: NC.M1.A-REI.6

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize in the next lesson.

Graphing technology should not be used in this cool-down.

Cool-down

Here is a system of linear equations: $\begin{cases} 2x + \frac{1}{2}y = 7 \\ 6x - \frac{1}{2}y = 5 \end{cases}$

1. Which would be more helpful for solving the system: adding the two equations or subtracting one from the other? Explain your reasoning.
2. Solve the system without graphing. Show your reasoning.

Student Reflection: Which method (graphing, substitution, elimination) of solving systems of equations makes you feel most confident in your math abilities? Which method makes you feel least confident? Please explain.

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

In the next lesson, students will continue to build their understanding of elimination. What do you notice in their work from today's lesson that you might leverage in the next lesson?

Practice Problems

1. Which equation is the result of adding these two equations? $\begin{cases} -2x+4y=17 \\ 3x-10y=-3 \end{cases}$

    a. $-5x-6y=14$
    b. $-x-6y=14$
    c. $x-6y=14$
    d. $5x+14y=20$

2. Which equation is the result of subtracting the second equation from the first? $\begin{cases} 4x-6y=13 \\ -5x+2y=5 \end{cases}$

    a. $-9x-4y=8$
    b. $-x+4y=8$
    c. $x-4y=8$
    d. $9x-8y=8$

3. Solve this system of equations without graphing: $\begin{cases} 5x+2y=29 \\ 5x-2y=41 \end{cases}$

4. Here is a system of linear equations: $\begin{cases} 6x+21y=103 \\ -6x+23y=51 \end{cases}$

    Would you rather use subtraction or addition to solve the system? Explain your reasoning.

5. Elena and Kiran are playing a board game. After one round, Elena says, "You earned so many more points than I did. If you earned 5 more points, your score would be twice mine!"

    Kiran says, "Oh, I don't think I did that much better. I only scored 9 points higher than you did."

    a. Write a system of equations to represent each student's comment. Be sure to specify what your variables represent.

    b. If both students were correct, how many points did each student score? Show your reasoning.

    (From Unit 3, Lesson 9)

6. Solve each system of equations by substitution.

    a. $\begin{cases} 2x+3y=4 \\ 2x=7y+24 \end{cases}$

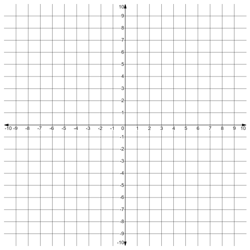
    b. $\begin{cases} 5x+3y=23 \\ 3y=15x-21 \end{cases}$

    (From Unit 3, Lesson 9)

7. A triangle has vertices $A(-6,-3)$, $N(6,5)$, and $T(2,-2)$.

    Is ANT isosceles? How do you know?

    (From Unit 3, Lesson 7)



8. Match each equation with the slope m and y-intercept of its graph.

Slope and y-intercept	Equation
a. $m = -6, y - int = (0, 12)$ b. $m = -6, y - int = (0, 5)$ c. $m = -\frac{5}{6}, y - int = (0, 1)$ d. $m = \frac{5}{6}, y - int = (0, 1)$ e. $m = \frac{5}{6}, y - int = (0, -1)$ f. $m = \frac{5}{6}, y - int = (0, -5)$	1. $5x - 6y = 30$ 2. $y = 5 - 6x$ 3. $y = \frac{5}{6}x + 1$ 4. $5x - 6y = 6$ 5. $5x + 6y = 6$ 6. $6x + y = 12$

(From Unit 3, Lesson 3)

9. Andre sells f full boxes and h half-boxes of fruit to raise money for a band trip. He earns $5 for each full box and $2 for each half-box of fruit he sells and earns a total of $100 toward the cost of his band trip. The equation $5f + 2h = 100$ describes this relationship.

Solve the equation for f.

(From Unit 2)

10. The volume of a cylinder is represented by the formula $V = \pi r^2 h$.

Find each missing height and show your reasoning.

Volume (cubic inches)	Radius (inches)	Height (inches)	Show your reasoning.
96π	4		
31.25π	2.5		
V	r		

(From Unit 2)

11. Select **all** the expressions that are equivalent to $5x + 30x - 15x$.

a. $5(x + 6x - 3x)$
b. $(5 + 30 - 15) \cdot x$
c. $x(5 + 30x - 15x)$
d. $5x(1 + 6 - 3)$
e. $5(x + 30x - 15x)$

(Addressing NC.6.EE.4)[1]

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Lesson 11: Solving Systems by Elimination (Part Two)

PREPARATION

Lesson Goals	Learning Targets
• Recognize that multiplying an equation by a factor creates an equivalent equation whose graph is the same as that of the original equation. • Solve systems of equations by multiplying one or both equations by a factor and then adding or subtracting the equations to eliminate a variable.	• I understand that multiplying each side of an equation by a factor creates an equivalent equation whose graph and solutions are the same as that of the original equation. • I can solve systems of equations by multiplying each side of one or both equations by a factor, then adding or subtracting the equations to eliminate a variable.

Lesson Narrative

This is the second lesson that develops the idea of solving systems of linear equations in two variables by elimination. Two new ideas are introduced here.

The first idea is that students can multiply one or both equations in the system by a factor to make it possible to eliminate a variable. Prior to this point, students worked only with systems where at least one variable in the equations had the same coefficient or opposite coefficients, making the variable removable when the equations were added or subtracted.

Here students see that this is not a requirement for a system to be solvable by elimination. They can first multiply one or both equations by a factor—chosen strategically so that the coefficients of one variable become equal or opposites. Then, the variable can be eliminated by adding an original equation and the new equation, or by subtracting one from the other.

In previous lessons, students wrote systems of linear equations to represent constraints. They used graphing to solve those systems. In the second activity of this lesson, students will write a system to represent a situation and solve using algebraic methods of substitution or elimination.

In what ways might this lesson give students opportunities to surprise you with their thinking or reasoning?

Focus and Coherence

Building On	Addressing
NC.M1.A-REI.1: Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning. **NC.M1.A-CED.3:** Create systems of linear equations and inequalities to model situations in context.	**NC.M1.A-REI.5:** Explain why replacing one equation in a system of linear equations by the sum of that equation and a multiple of the other produces a system with the same solutions. **NC.M1.A-REI.6:** Use tables, graphs, or algebraic methods (substitution and elimination) to find approximate or exact solutions to systems of linear equations and interpret solutions in terms of a context.

Agenda, Materials, and Preparation

- **Warm-up** (*10 minutes*)
  - Technology is required for this lesson: Acquire devices that can access Desmos (recommended) or other spreadsheet technology. It is ideal if each student has their own device. Be prepared to display a graph using technology for all to see.
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
- **Activity 3** (*Optional: 15 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U3.L11 Cool-down (print 1 copy per student)

LESSON

Warm-up: Multiplying Equations by a Factor (*10 minutes*)

Instructional Routine: Graph It	
Building On: NC.M1.A-REI.1	**Addressing:** NC.M1.A-REI.6

In this *Graph It* warm-up, students will experiment with the graphical effects of multiplying both sides of an equation in two variables by a factor.

Students are prompted to multiply one equation in a system by several factors to generate several equivalent equations. They then graph these equations on the same coordinate plane that shows the graphs of the original system. Students notice that no new graphs appear on the coordinate plane and reason about why this might be the case.

The work here reminds students that equations that are equivalent have all the same solutions, so their graphs are also identical. Later, students will rely on this insight to explain why they can multiply one equation in a system by a factor—which produces an equivalent equation—and solve a new system containing that equation instead.

Step 1

- Display the equation $x + 5 = 11$ for all to see. Ask students:
  - "What is the solution to this equation?" ($x = 6$)
  - "If we multiply both sides of the equation by a factor, say, 4, what equation would we have?" ($4x + 20 = 44$) "What is the solution to this equation?" ($x = 6$)
  - "What if we multiply both sides by 100?" ($100x + 500 = 1,100$) "or by 0.5?" ($0.5x + 2.5 = 5.5$)
  - "Is the solution to each of these equations still $x = 6$?" (Yes)

- Remind students that these equations are one-variable equations, and that multiplying both sides of a one-variable equation by the same factor produces an equivalent equation with the same solution.
- Ask students: "What if we multiply both sides of a two-variable equation by the same factor? Would the resulting equation have the same solutions as the original equation?" Tell students that they will now investigate this question by graphing.

Step 2

- Ask students to arrange themselves into groups of three or use visibly random grouping.
- Provide access to Desmos or other graphing technology to each group.
- To save time, ask group members to divide up the tasks. (For example, one person could be in charge of graphing while the others write equivalent equations, and everyone analyzes the graphs together.)

Student Task Statement

Consider two equations in a system:

$$\begin{cases} 4x + y = 1 & \text{Equation A} \\ x + 2y = 9 & \text{Equation B} \end{cases}$$

1. Use graphing technology to graph the equations. Then, identify the coordinates of the solution.
2. Write equations that are equivalent to equation A by multiplying both sides of it by the same number, for example, 2 or -5. Let's call the resulting equations A1 and A2. Record your equations here:
    a. Equation A1: __________
    b. Equation A2: __________
3. Using technology, graph the equations you generated. Make a couple of observations about the graphs.

Step 3

- Invite students to share their observations about the graphs they created. Ask students why the graphs of equations A1 and A2 all coincide with the graphs of the original equation A. Discuss with students:
    - "How can we explain the identical graphs?" (Equations A1 and A2 are equivalent to equation A. They both have the same solutions as equation A, and their graphs are the same line as the graph of A.)
    - "What move was made to generate A1 and A2? Why did it create equations that are equivalent to A?" (The two sides of equation A were multiplied by the same factor, which keeps the two sides equal.)
- If time allows, remind students that, earlier in the unit, they saw that isolating one variable is a way to see if two equations are equivalent. If we isolate y in equations A, A1, and A2 the rearranged equation will be identical: $y = -4x + 1$.

DO THE MATH	PLANNING NOTES

Activity 1: Writing a New System to Solve a Given System (*15 minutes*)

Instructional Routine: Critique, Correct, Clarify (MLR3)	
Building On: NC.M1.A-REI.1	**Addressing:** NC.M1.A-REI.5; NC.M1.A-REI.6

In the previous lesson, students learned that adding two equations in a system creates a new equation that shares a solution with the system. In the warm-up, they saw that multiplying an equation by a factor creates an equivalent equation that shares all the same solutions as the original equation.

Here students learn that each time we perform a move that creates one or more new equations, they are in fact creating a new system that is equivalent to the original system. Equivalent systems are systems with the same solution set, and writing a series of equivalent systems can help you get closer to finding the solution of an original system.

For instance, if $4x + y = 1$ and $x + 2y = 9$ form a system, and $4x + 8y = 36$ is a multiple of the second equation, the equations $4x + y = 1$ and $4x + 8y = 36$ form an equivalent system that can help us eliminate x and make progress toward finding the value of y.

Students also learn to make an argument that explains why each new system is indeed equivalent to the one that came before it (MP3), building on the work of justifying equivalent equations in earlier lessons.

Step 1

- Keep students in groups of three.
- Give students 2–3 minutes of quiet work time.
- Provide students 1–2 minutes to discuss their thinking with their group.

RESPONSIVE STRATEGY

Provide the steps for finding the solution (-1,5) but share them out of order. Have students place them in the correct order as an alternative to completing question 2.

Advancing Student Thinking: Some students may think that the second step should have resulted in $9y = 37$. Remind students that they are subtracting every term in the second equation, which means $y - 8y$ and $1 - 36$, or they are distributing the -1 in the second equation and combining the two equations.

Student Task Statement

Here is a system you solved by graphing earlier.

$$\begin{cases} 4x + y = 1 & \text{equation A} \\ x + 2y = 9 & \text{equation B} \end{cases}$$

To start solving the system, Elena wrote:

$$\begin{array}{c} 4x + y = 1 \\ x + 2y = 9 \end{array} \longrightarrow \begin{array}{c} 4x + y = 1 \\ 4x + 8y = 36 \end{array}$$

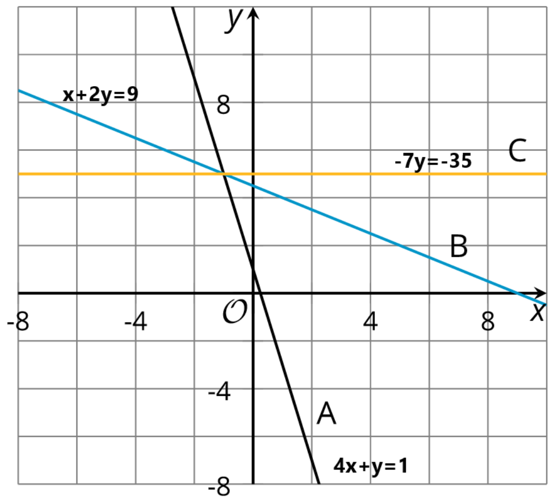
And then she wrote:

$$\begin{array}{c} 4x + y = 1 \\ 4x + 8y = 36 \end{array} \longrightarrow \begin{array}{rr} & 4x + y = 1 \\ - & (4x + 8y = 36) \\ \hline & -7y = -35 \end{array}$$

1. What were Elena's first two moves? What might be possible reasons for those moves?
2. Complete the solving process algebraically. Show that the solution is indeed $x = -1, y = 5$.

3. Elena wanted to check her solution from a different perspective. She graphed the original equations of the system $4x + y = 1$, $x + 2y = 9$, as well as the equation she got by combining the two equations: $-7y = -35$.

What do you notice? What do you wonder?



Step 2

- Begin a brief whole-class discussion by telling students: "One way to create an equivalent system is by multiplying one or both equations by a factor. It helps to choose the factor strategically." Then ask, "How can you identify the factor that will create an equivalent system where elimination can be used to solve?" (identify the common multiple between the two coefficients and determine which factor can be used to get that multiple)

Tell students that they were asked to summarize Elena's moves for a student who was absent. Use the *Critique, Correct, Clarify* routine by providing students with the following first draft writing of a summary to improve:

"It is not surprising that $(-1,5)$ is the solution to Elena's new system, just because her method eliminated x by first multiplying to rewrite one equation.Then she subtracted something the same from both sides of an equation with that solution, so her method didn't really change anything, even the solution."

- Give students 1 minute to first identify any parts of the first draft that could be more clear or complete (or correct). Display the first draft writing, and spend 1–2 minutes having two or three students share ideas about what could be improved. As students share, annotate the displayed writing by circling, underlining and marking it with arrows and labels (e.g., "clarify," "add detail," "?" etc.).
- Ask students (individually or in pairs) to spend 1–2 minutes writing a second draft that is more clear and more complete than the first draft. Circulate and look for students who add clarity to the ideas of equivalence, multiplying by a factor, choosing a factor, eliminating a variable, subtracting the same amount from both sides, and having the same solution.
- Invite one or two students to read their second drafts aloud. Spend 3–5 minutes engaging in "live editing" to generate a third draft by scribing slowly as each student reads their draft aloud. While scribing is happening, invite the author of each draft, and the rest of the class as well, to offer revised wording and additional details.

DO THE MATH	PLANNING NOTES

Activity 2: Classroom Supplies (*10 minutes*)

Instructional Routine: Three Reads (MLR6)	
Building On: NC.M1.A-CED.3	**Addressing:** NC.M1.A-REI.5, NC.M1.A-REI.6

In a previous lesson, students created a system of linear equations to model constraints in a situation. In this activity, students will write the system, solve the system, and interpret the solution in context.

Step 1

- Ask students to arrange themselves into pairs or use visibly random grouping.
- Use the *Three Reads* routine to get students started on problem 1.
  - First Read: Read the problem aloud to the class. "A teacher purchased 20 calculators and 10 measuring tapes for her class and paid $495. Later, she realized that she didn't order enough supplies. She placed another order of eight of the same calculators and one more of the same measuring tape and paid $178.50."
    - Ask students: "What is this situation about? What is going on here?"
    - Spend less than 1 minute scribing student ideas. Let students know the focus is just on the situation, not on the numbers. (For example, students might say "a teacher buys calculators and measuring tapes for her class.")
  - Second Read: Display the situation and ask a student volunteer to read it aloud to the class again.
    - Then ask: "What are the quantities in this situation? A quantity is something that can be counted or measured."
    - Again, spend less than a minute scribing student responses. Listen for, and amplify, the important quantities that vary in relation to each other in this situation: number of calculators purchased (for each order), number of measuring tapes purchased (for each order), and total cost of each order.
  - Third Read: Invite students to read the situation again to themselves, or ask another student volunteer to read it aloud. After the third read, reveal the first question that follows.

RESPONSIVE STRATEGY

Provide appropriate reading accommodations and supports to ensure student access to written directions, word problems and other text-based content. Pause in between sentences and reread or repeat gestures if necessary for increased comprehension.

Supports accessibility for: Language; Conceptual processing

- Ask, "How might we approach the question being asked? What is the first thing you will do?" (write two equations each representing one of the orders)
- Spend 1–2 minutes scribing student ideas as they brainstorm possible starting points. Be sure to stop any students who begin to share a complete solution; the goal is to crowdsource some starting points. This routine helps students interpret the language within a given situation needed to create a system of linear equations.

- Provide students 2–3 minutes of quiet time to write and solve a system of linear equations and a few minutes to discuss their thinking around question 3 with their partner.

Monitoring Tip: Monitor for how students solve the system of linear equations: elimination or substitution. Identify students who use each strategy and let them know that they may be asked to share later. Intentionally seek to highlight approaches from students who do not typically volunteer.

Advancing Student Thinking: Students may stop solving when they have found the value of one of the variables. For example, if solving by elimination they may stop once they determine that $c = 21.50$. Remind students that a solution to a system of equations in two variables is a pair of values.

Student Task Statement

A teacher purchased 20 calculators and 10 measuring tapes for her class and paid $495. Later, she realized that she didn't order enough supplies. She placed another order of eight of the same calculators and one more of the same measuring tape and paid $178.50.

1. Write a system of equations representing the constraints in the situation. Let c represent the price of a calculator and m represent the price of a measuring tape.
2. Solve the system of equations without graphing. Explain or show your reasoning.
3. What does the solution to the system mean in this situation?

Step 2

- Invite students previously selected to share their strategy and solution. Ask students "What does the solution mean in this situation?" (The price of the calculator and price of the measuring tape that make the equations for both purchases true.)

DO THE MATH	PLANNING NOTES

Activity 3: Build Some Equivalent Systems (*Optional, 15 minutes*)

Addressing: NC.M1.A-REI.5, NC.M1.A-REI.6

This optional activity gives students another opportunity to identify moves that lead to equivalent systems. It also prompts students to build their own equivalent systems, which encourages them to think strategically about what to do to reach the goal of solving the system, rather than focusing solely on a particular solution path.

As students work, prompt students to articulate the reasoning and assumptions. Ask questions such as:

- "How do you choose the factor to use so a variable can be eliminated?"
- "Can the factor be a fraction? Why or why not?"(Yes, multiply the second equation by $\frac{1}{2}$.) “A negative number?” (Yes, multiply the second equation by -3.) “Zero?” (No, that would eliminate the entire equation.)

Step 1

- Keep students in pairs to complete problems 1–5 if observations from Activity 2 provide evidence of needs for some to have peer collaborative support. Otherwise, have students complete this activity independently for individual formative assessment.

Advancing Student Thinking: If students struggle to create an equivalent system of their own, ask them to start by deciding on a variable they'd like to eliminate. Then, ask them to think about a factor that, when multiplied to one equation, would produce the same or opposite coefficients for that variable.

Student Task Statement

Here is a system of equations:

$$\begin{cases} 6x + 5y = -7 \\ 2x - 10y = -14 \end{cases}$$

To solve this system, Diego wrote the following equivalent system:

$$\begin{array}{rcr} 6x + 5y = -7 & & 12x + 10y = -14 \\ 2x - 10y = -14 & \longrightarrow & +\quad (2x - 10y = -14) \\ \hline & & 14x = -28 \end{array}$$

1. Describe the moves that Diego made to create the equivalent system and eliminate the y-variable.
2. Write another equivalent system (different than Diego’s) that will allow one variable to be eliminated and enable you to solve the original system. Be prepared to describe the moves you make to create the equivalent system.
3. Use your equivalent system to solve the original system. Then, check your solution by substituting the pair of values into the original system.

Step 2

- Invite students with different first steps to display their equivalent systems and solution paths. Ask students to verify that, regardless of the moves made, the different paths all led to the same pair of values.
- If no one mentions dividing the second equation by 2 (or multiplying by $\frac{1}{2}$), display this as a possible first step in creating an equivalent system. Ask students what would be the advantage of this approach as opposed to another.
- Listen for and amplify students' use of "equivalent" as they verify their work.

RESPONSIVE STRATEGY

Support students to verify their work with word wall: system, equivalent, substitute, eliminate, value.

DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

In this lesson students developed a strategy for creating an equivalent system for solving by elimination. Facilitate a discussion using the following questions. As students respond, make connections to examples from the lesson.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

Display the following systems:

System 1

$$\begin{cases} 4x + 3y = 10 \\ -4x + 5y = 6 \end{cases}$$

System 2

$$\begin{cases} 2x + 3y = 16 \\ 6x - 5y = 20 \end{cases}$$

- "What moves are needed to solve each system by elimination?" (For system 1, the two equations can be added to eliminate the x-variable making it possible to solve for y. For system 2, multiply the first equation by 3 then subtract the two equations. This will eliminate the x-variable making it possible to solve for y.)
- "What do you look for in a system to determine if elimination can be used to solve?" (coefficients of the variables that are either the same or opposite)
- "How can you identify the factor that will create an equivalent system where elimination can be used to solve?" (identify the common multiple between the two coefficients and determine which factor can be used to get that multiple)

PLANNING NOTES

Student Lesson Summary and Glossary

We now have two algebraic strategies for solving systems of equations: substitution and elimination. In some systems, the equations may give us a clue as to which strategy to use. For example:

$$\begin{cases} y = 2x - 11 \\ 3x + 2y = 18 \end{cases}$$

In this system, y is already isolated in one equation. We can solve the system by substituting $2x-11$ for y in the second equation and finding x.

$$\begin{cases} 3x - y = -17 \\ -3x + 4y = 23 \end{cases}$$

This system is set up nicely for elimination because of the opposite coefficients of the x-variable. Adding the two equations eliminates x so we can solve for y.

In other systems, which strategy to use is less straightforward, either because no variables are isolated, or because no variables have equal or opposite coefficients. For example:

$$\begin{cases} 6x - 12y = 24 & \text{equation A} \\ -x + 3y = -5 & \text{equation B} \end{cases}$$

To solve this system by elimination, we first need to rewrite one or both equations so that one variable can be eliminated. To do that, we can multiply each side of an equation by the same factor. Remember that doing this doesn't change the equality of the two sides of the equation, so the x- and y-values that make the first equation true also make the new equation true.

There are different ways to eliminate a variable with this approach. For instance, we could:

Multiply equation B by 6 to get $-6x + 18y = -30$. Adding the equations of the equivalent system will result in eliminating the x variable.

$$\begin{array}{c} 6x - 12y = 24 \\ -x + 3y = -5 \end{array} \longrightarrow \begin{array}{c} 6x - 12y = 24 \\ -6x + 18y = -30 \end{array}$$

Multiply equation B by 4 to get $-4x + 12y = -20$. Adding the equations of the equivalent system will result in eliminating the y-variable.

$$\begin{array}{c} 6x - 12y = 24 \\ -x + 3y = -5 \end{array} \longrightarrow \begin{array}{c} 6x - 12y = 24 \\ -4x + 12y = -20 \end{array}$$

Multiply equation A by $\frac{1}{6}$ to get $x - 2y = 4$. Adding the equations of the equivalent system will result in eliminating the x-variable.

$$\begin{array}{c} 6x - 12y = 24 \\ -x + 3y = -5 \end{array} \longrightarrow \begin{array}{c} x - 2y = 4 \\ -x + 3y = -5 \end{array}$$

Each multiple of an original equation is equivalent to the original equation. So each new pair of equations is an equivalent system to the original system and has the same solution.

Let's solve the original system using the first equivalent system we found earlier.

Adding the equations eliminates the x, leaving a new equation $6y = -6$ or $y = -1$.

$$\begin{array}{c} 6x - 12y = 24 \\ -x + 3y = -5 \end{array} \longrightarrow \begin{array}{rr} & 6x - 12y = 24 \\ + & (-6x + 18y = -30) \\ \hline & 6y = -6 \\ & y = -1 \end{array}$$

Substituting -1 for y in the second equation allows us to solve for x.

$$\begin{aligned} -x + 3(-1) &= -5 \\ -x - 3 &= -5 \\ -x &= -2 \\ x &= 2 \end{aligned}$$

Cool-down: Make Your Move (*5 minutes*)

Building Towards: NC.M1.A-REI.6

Cool-down Guidance: More Chances
If students were not able to explain why the moves both created new equations with the same solutions as the original equation, consider examining the process of solving the equation by using both methods (either whole-class or in partners). While many students will find multiplying the equations by 3 to be less prone to errors (due to eliminating the fraction), it is important for students to know that both moves are acceptable and will yield the correct solution.

In each case, the moves are acceptable because multiplying each side of an equation by a factor preserves the equality. If students are having trouble seeing this, consider writing the equation $1+3=4$ and examine what happens when each side is multiplied by 3.

Cool-down

Lin and Priya were working on solving this system of equations.

$$\begin{cases} \frac{1}{3}x + 2y = 4 \\ x + y = -3 \end{cases}$$

Lin's first move is to multiply the first equation by 3.

Priya's first move is to multiply the second equation by 2.

1. Explain why either move creates a new equation with the same solutions as the original equation.
2. Whose first move would you choose to do to solve the system? Explain your reasoning.

Student Reflection:

When it comes to math practice (*fill in the blank*) ___________ because ________________________________.

a. I practice often b. I practice sometimes c. I practice very seldomly c. I do not practice

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Reflect on a time your thinking changed about something in class recently. How will you alter your teaching practice to incorporate your new understanding?

Practice Problems

1. Solve each system of equations.

   a. $\begin{cases} 2x - 4y = 10 \\ x + 5y = 40 \end{cases}$

   b. $\begin{cases} 3x - 5y = 4 \\ -2x + 6y = 18 \end{cases}$

2. Tyler is solving this system of equations: $\begin{cases} 4p + 2q = 62 \\ 8p - q = 59 \end{cases}$

   He can think of two ways to eliminate a variable and solve the system:

   - Multiply $4p + 2q = 62$ by 2, then subtract $8p - q = 59$ from the result.
   - Multiply $8p - q = 59$ by 2, then add the result to $4p + 2q = 62$.

   Do both strategies work for solving the system? Explain or show your reasoning.

3. Andre and Elena are solving this system of equations: $\begin{cases} y = 3x \\ y = 9x - 30 \end{cases}$

   Andre's first step is to write: $3x = 9x - 30$

   Elena's first step is to create a new system: $\begin{cases} 3y = 9x \\ y = 9x - 30 \end{cases}$

   Do you agree with either first step? Explain your reasoning.

4. Select **all** systems that are equivalent to this system: $\begin{cases} 6d + 4.5e = 16.5 \\ 5d + 0.5e = 4 \end{cases}$

   a. $\begin{cases} 6d + 4.5e = 16.5 \\ 45d + 4.5e = 4 \end{cases}$

   b. $\begin{cases} 30d + 22.5e = 82.5 \\ 5d + 0.5e = 4 \end{cases}$

   c. $\begin{cases} 30d + 22.5e = 82.5 \\ 30d + 3e = 24 \end{cases}$

   d. $\begin{cases} 6d + 4.5e = 16.5 \\ 6d + 0.6e = 4.8 \end{cases}$

   e. $\begin{cases} 12d + 9e = 33 \\ 10d + 0.5e = 8 \end{cases}$

   f. $\begin{cases} 6d + 4.5e = 16.5 \\ 11d + 5e = 20.5 \end{cases}$

5. Here is a system of equations with a solution: $\begin{cases} p + 8q = -8 \\ \frac{1}{2}p + 5q = -5 \end{cases}$

   a. Write a system of equations that is equivalent to this system. Describe what you did to the original system to get the new system.

   b. Explain how you know the new system has the same solution as the original system.

6. Here is a system of equations: $\begin{cases} -7x + 3y = -65 \\ -7x + 10y = -135 \end{cases}$

   Write an equation that results from subtracting the two equations.

   (From Unit 3, Lesson 10)

7. Here is a system of linear equations: $\begin{cases} 2x + 7y = 8 \\ y + 2x = 14 \end{cases}$

   Find at least one way to solve the system by substitution and show your reasoning. How many ways can you find? (Regardless of the substitution that you do, the solution should be the same.)

   (From Unit 3, Lesson 9)

8. A grocery store sells bananas for b dollars per pound and grapes for g dollars per pound. Priya buys 2.2 pounds of bananas and 3.6 pounds of grapes for \$9.35. Andre buys 1.6 pounds of bananas and 1.2 pounds of grapes for \$3.68.

   Write a system of equations to represent the constraints in this situation.

   (From Unit 3, Lesson 9)

9. Noah wants to mail a package to his friend and the cost of mailing it is \$5.00. Noah asked his mom for \$5.00 but instead she gave him some postcard stamps that are worth \$0.34 each and some first-class stamps that are worth \$0.49 each.

   a. Write an equation that relates the number of postcard stamps , p , the number of first-class stamps, f , and the cost of mailing the package.

   b. Solve the equation for f .

   c. Solve the equation for p .

   d. If Noah puts seven first-class stamps on the package, how many postcard stamps will he need?

   (From Unit 2)

Lesson 12: Systems of Linear Equations and Their Solutions

PREPARATION

Lesson Goals	Learning Targets
• Determine whether a system of equations will have no solutions, one solution, or infinitely many solutions by analyzing their structure or by graphing. • Recognize that a system of linear equations can have 0, 1, or infinitely many solutions. • Use the structure of the equations in a linear system to make sense of the properties of their graphs.	• I can tell how many solutions a system has by graphing the equations or by analyzing the parts of the equations and considering how they affect the features of the graphs. • I know the possibilities for the number of solutions a system of equations could have.

Lesson Narrative

This lesson serves two main goals. The first goal is to revisit the idea (first learned in middle school) that not all systems of linear equations have a single solution. Some systems have no solutions, and others have infinitely many solutions. The second goal is to investigate different ways to determine the number of solutions to a system of linear equations.

Earlier in the unit, students learned that the solution to a system of equations is a pair of values that meet both constraints in a situation, and that this condition is represented by a point of intersection of two graphs. Here, students make sense of a system with no solutions in a similar fashion. They interpret it to mean that there is no pair of values that meet both constraints in a situation, and that there is no point at which the graph of each equation would intersect.

Next, students use what they learned about the structure of equations and about equivalent equations to reason about the number of solutions. This means that if the two equations in a system are equivalent, they can tell—without graphing—that the system has infinitely many solutions. These exercises are opportunities to look for and make use of structure (MP7). Likewise, students are aware that the graphs of linear equations with the same slope but different vertical intercepts are parallel lines and therefore have no solutions. If the equations in a system can be rearranged into slope-intercept form (where the slope and vertical intercept become "visible"), it is possible to determine how many solutions a system has without graphing.

In what ways will you encourage students to persevere in this lesson?

Focus and Coherence

Building On	Addressing
NC.8.EE.8: Analyze and solve a system of two linear equations in two variables in slope-intercept form. • Understand that solutions to a system of two linear equations correspond to the points of intersection of their graphs because the point of intersection satisfies both equations simultaneously. • Solve real-world and mathematical problems leading to systems of linear equations by graphing the equations. Solve simple cases by inspection. **NC.M1.A-CED.4:** Solve for a quantity of interest in formulas used in science and mathematics using the same reasoning as in solving equations.	**NC.M1.A-CED.3:**Create systems of linear equations and inequalities to model situations in context. **NC.M1.A-REI.6:** Use tables, graphs, or algebraic methods (substitution and elimination) to find approximate or exact solutions to systems of linear equations and interpret solutions in terms of a context.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*10 minutes*)
  - Technology is required for this lesson: Acquire devices that can access Desmos (recommended) or other graphing technology. It is ideal if each student has their own device.
- **Activity 2** (*15 minutes*)
  - Sorting Systems card sort (print 1 copy per group of students and cut up in advance)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U3.L12 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.8.EE.8

In grade 8, students learned that some linear equations have one, infinitely many, or no solutions. This bridge reminds students about this fact, while also prompting them to think about how structure can help them better understand the number(s) of solutions an equation has. This task is aligned to question 4 in Check Your Readiness.

Student Task Statement

Of the three equations below, one has one solution, one has no solutions, and one has infinitely many solutions. Match each equation with the number of solutions it has.[1]

1. $12(x-3)+18=6(2x-3)$
2. $12(x-3)+18=4(3x-3)$
3. $12(x-3)+18=4(2x-3)$

a. one solution
b. no solutions
c. infinitely many solutions

[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

DO THE MATH	PLANNING NOTES

Warm-up: Graphs of Systems (*5 minutes*)

Instructional Routine: Which One Doesn't Belong?
Addressing: NC.M1.A-REI.6

This warm-up prompts students to carefully analyze and compare systems of linear equations with different numbers of solutions using the *Which One Doesn't Belong?* routine. In making comparisons, students have a reason to use language precisely (MP6). They may also recognize and make use of the structure of the solution(s) revealed in the graph (MP8).

The work here prepares students to recognize what no solution, one solution, and infinitely many solutions looks like both in equation form and graphically.

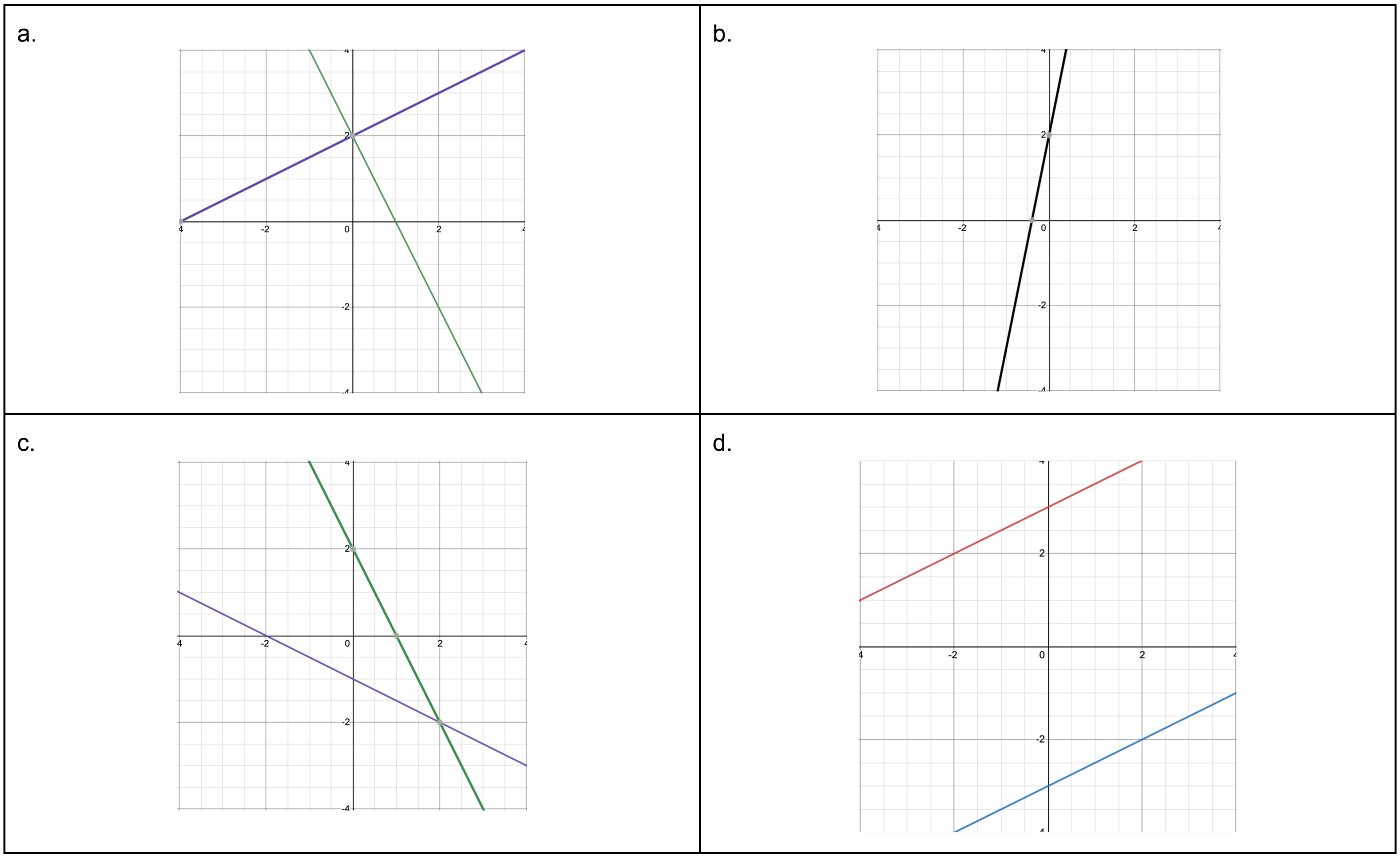
Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Display the graphs for all to see.
- Give students 1 minute of quiet think time followed by 1 minute to share their thinking with their partner.
  - Prompt each student to share with their partner their reasoning for why a particular graph does not belong.
  - After both partners have shared, if time permits, ask them to work together to find a reason each item doesn't belong.

Student Task Statement

Which one doesn't belong? Explain your reasoning.

a. b. c. d.



Step 2

- Ask a few partners to share one reason why a particular item does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree and if anyone has a different reason to share for the same item. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given are correct.
- During the discussion, ask students how they see the "solution(s) or no solution" in each of the graphs.
- Additionally, ask students to explain what they know about the slopes of a system with no solution, what they might know about the slope and y-intercept of the system with infinite solutions, what they would know about the slope and y-intercept of the system with one solution.

DO THE MATH

PLANNING NOTES

Activity 1: What's the Deal? (*10 minutes*)

Instructional Routines: Graph It; Three Reads (MLR6)	
Building On: NC.8.EE.8	**Addressing:** NC.M1.A-CED.3, NC.M1.A-REI.6.

In this *Graph It* activity, students encounter a contextual situation that can be represented with a system of equations but the system has no solutions. Students write equations to represent the two constraints in the situation and then solve the system algebraically and graphically.

Step 1

- Keep students in pairs and provide access to graphing technology.
- Use the *Three Reads* routine to support comprehension of this word problem.
  - First Read: Without displaying the task, read the context aloud to the class: "A recreation center is offering special prices on its pool passes and gym memberships for the summer. On the first day of the offering, a family paid $96 for four pool passes and two gym memberships. Later that day, an individual bought a pool pass for herself, a pool pass for a friend, and one gym membership. She paid $72."
    - Ask students: "What is this situation about? What is going on here?"
    - Let students know the focus is just on the situation, not on the numbers. (For example, students might say, "it's about a recreation center that sells pool passes and gym memberships" or "it's about people buying passes and memberships.")
    - Spend less than a minute scribing their understanding of the situation in a place where all can see. Do not correct students, but do clarify any unfamiliar words; visuals often help (for example, a picture of a pool and gym).
  - Second Read: Display the task context without the problems, and ask a student volunteer to read it aloud to the class again.
    - Ask: "What are the quantities in this situation? A quantity is something that can be counted or measured." (for example, total amount of money paid by the family and by the individual, the number of pool passes, and number of gym memberships purchased by each).
    - Again, spend less than a minute scribing student responses as they brainstorm as many quantities as they can think of in a short amount of time.
    - Encourage students to identify quantities that are named in the problem explicitly, and any quantities that may be implicit. For each quantity (for example, "72"), ask students to add details (for example, "the total amount the individual spent was $72").
  - Third Read: Invite students to read the task context and questions 1–3 to themselves, or ask another student volunteer to read these aloud.
    - Ask: "How might we approach the questions being asked? What is the first thing you will do?"
    - Spend 1–2 minutes scribing student ideas as they brainstorm possible starting points.
    - Be sure to stop any students who begin to share a complete solution; the goal is to crowdsource some starting points.

Step 2

- Provide students 1–2 minutes of quiet work time on questions 1–2 before comparing responses with their partner.
- Have students continue to work together to complete question 3.

Monitoring Tip: As students work, notice the different ways they reach the conclusion that the systems have no solutions. Identify students who use each strategy and let them know that they may be asked to share later. Include at least one student who does not typically volunteer.

Advancing Student Thinking**:** If students are concerned that they are doing something wrong because their algebraic methods do not yield a solution, encourage them to go on to question 3, graphing the equations, to see what is going on.

Student Task Statement

A recreation center is offering special prices on its pool passes and gym memberships for the summer. On the first day of the offering, a family paid $96 for four pool passes and two gym memberships. Later that day, an individual bought a pool pass for herself, a pool pass for a friend, and one gym membership. She paid $72.

1. Write a system of equations that represents the relationships between pool passes, gym memberships, and the costs. Be sure to state what each variable represents.
2. Find the price of a pool pass and the price of a gym membership by solving the system algebraically. Explain or show your reasoning.
3. Use graphing technology to graph the equations in the system. Make 1–2 observations about your graphs.

Step 3

- Select identified students to share how they solved this system algebraically. Record or display their reasoning for all students to see. Ask if anyone else reasoned the same way.
- Next, select other students to share their observations about the graphs. Ask students:
  - “How can we tell from the graphs that there are no solutions?” (The lines are parallel.)
  - “How can we tell for sure that the lines are parallel and never intersect?” (The slope of both graphs is -2 but they have different intercepts.)
  - “Why do parallel lines mean no solutions?” (A solution is a pair of values that satisfy both equations and are on both graphs. There are no points that are on both lines simultaneously.)
  - “What does ‘no solutions’ mean in this situation, in terms of price of pool passes and gym memberships?” (The prices for a pool pass and for a gym membership are different for the two purchases.)
  - Ask students for some realistic reasons why this scenario does not have a solution. Here’s a few to consider:
    - The family purchased twice the number of of pool passes and gym memberships as the individual did, but they did not pay twice as much, so the prices of passes and memberships must have been different for the two purchases.
    - The person who bought half as many passes and memberships did not pay half as much, which meant that different prices applied to the two transactions.
    - The special rates for a family of four did not apply to individuals, hence the different prices.

DO THE MATH	PLANNING NOTES

Activity 2: Sorting Systems (*15 minutes*)

Instructional Routines: Card Sort; Discussion Supports (MLR8) - Responsive Strategy	
Building On: NC.M1.A-CED.4	**Addressing:** NC.M1.A-REI.6

In this *Card Sort* activity, students apply the structure of equations to sort systems of equations based on the number of solutions (one solution, many solutions, or no solutions).

Students could solve each system algebraically or graphically and sort afterwards, but given the number of systems to be solved, they will likely find this process to be time consuming. Encourage students to make use of the structure in the equations in the systems (MP7).

Since the purpose of this activity is for students to analyze the structure of equations in systems, technology is not an appropriate tool.

CARD SORT	***What Is This Routine?*** A *Card Sort* uses cards or slips of paper that can be manipulated and moved around (or the same functionality enacted with a computer interface). It can be done individually or in small groups. Students put things into categories or groups based on shared characteristics or connections. This routine can be combined with *Take Turns*, such that each time a student sorts a card into a category or makes a match, they are expected to explain the rationale while the group listens for understanding. The first few times students engage in these activities, the teacher should demonstrate how the activity is expected to go. Once students are familiar with these structures, less set-up will be necessary. While students are working, the teacher can ask students to restate their question more clearly or paraphrase what their partner said. ***Why This Routine?*** A *Card Sort* provides opportunities to attend to mathematical connections using representations that are already created, instead of expending time and effort generating representations. It gives students opportunities to analyze representations, statements, and structures closely and make connections (MP2, MP7).

Here is an image of the cards for reference and planning:

Sorting Systems Card 1 $\begin{cases} y = 2x - 7 \\ y = -7x + 2 \end{cases}$	Sorting Systems Card 2 $\begin{cases} y = 2x - 3 \\ y = 2x - 13 \end{cases}$	Sorting Systems Card 3 $\begin{cases} y = -\frac{1}{3}x - 3 \\ 3y = -9 - x \end{cases}$	Sorting Systems Card 4 $\begin{cases} 3x + y = -10 \\ 3y = -x - 10 \end{cases}$
Sorting Systems Card 5 $\begin{cases} x - 4y = -12 \\ 5x - 20y = 60 \end{cases}$	Sorting Systems Card 6 $\begin{cases} x - y = -6 \\ x - 4y = 12 \end{cases}$	Sorting Systems Card 7 $\begin{cases} 4y = x + 4 \\ y = \frac{1}{4}x + 1 \end{cases}$	Sorting Systems Card 8 $\begin{cases} x + y = 5 \\ x + y = 12 \end{cases}$

Step 1

- Keep students in pairs.
- Tell students they will be sorting a set of cards by the number of solutions in the system: one, infinitely many, or no solutions.
- Remind students to use the structure of the equations in the system instead of graphing (MP7). Examples of structure include looking for equivalent equations, equations with the same slope but different vertical intercepts, variable expressions with the same or opposite coefficients, and so on. Some equations may need to be rearranged with close attention to all parts - the signs, variables, coefficients and constants (MP6).
- Give one set of cards to each group.pre-cut slips or cards from the blackline master to each group.
- Give students 7–8 minutes to sort the cards into groups. Emphasize to students that they should be prepared to explain how they placed each system. Follow with a whole-class discussion.

Monitoring Tip: As students discuss their thinking in groups, make note of the different ways they use structure to complete the task. Encourage students who are solving individual systems to analyze the features of the equations and see if they could reason about the solutions or gain information about the graphs that way.

RESPONSIVE STRATEGY

Use this routine to support small-group discussion during the card sort. Encourage students to take turns selecting a card and to explain to their partner whether the system has no solutions, one solution, or infinitely many solutions. Display the following sentence frames for all to see: "This system has ____ solutions because...." Encourage students not only to challenge each other if they disagree, but also to press for clear explanations that use mathematical language.

Discussion Supports (MLR8)

RESPONSIVE STRATEGY

Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches. Be sure to include at least one card from each category in the initial set.

Supports accessibility for: Conceptual processing; Organization

Advancing Student Thinking: Some students may not know how to begin sorting the cards. Suggest that they try solving one or two systems. Ask them to notice if there's a point in the solving process when they realize how many solutions the system has or what the graphs of the two equations would look like. Encourage them to look for similarities in the structure of the equations and see how the structure might be related to the number of solutions.

Student Task Statement

Your teacher will give you a set of cards. Each card contains a system of equations.

Sort the systems into three groups based on the number of solutions each system has. Be prepared to explain how you know where each system belongs.

Are You Ready For More?

1. In the cards, for each system with no solution, change a single constant term so that there are infinitely many solutions to the system.
2. For each system with infinitely many solutions, change a single constant term so that there are no solutions to the system.
3. Explain why in these situations it is impossible to change a single constant term so that there is exactly one solution to the system.

Step 2

- Invite groups to share their sorting results and record them. Ask the class if they agree or disagree. If there are disagreements, ask students who disagree to share their reasoning.
- Display all the systems—sorted into groups—for all to see and discuss the characteristics of the equations in each group. Ask students questions such as:
  - "How can we tell from looking at the equations in cards 2, 5, and 8 that the systems have no solution?" Possible reasoning:
    - Card 2: The equations are in slope-intercept form. The slope is 2 for both graphs, but the y-intercept is different, so the lines must be parallel.
    - Card 5: The second equation can be multiplied by $\frac{1}{5}$ to give $x–4y = -12$. Both equations now have $x–4y$ on one side, but that expression is equal to 4 in the first equation and equal to -4 in the second. There is no pair of x and y that can make both equations true.
    - Card 8: There is no pair of x and y where $x + y$ can simultaneously equal both 5 and 12.
  - "What about the equations in cards 3 and 7? What features might give us a clue that the systems have infinitely many solutions?" Possible reasoning:
    - Card 3: The coefficients and constants in the second equation are 3 times those in the first, so they are equivalent equations.
    - Card 7: The equations have the same slope and y-intercept, resulting in all the same solutions.
  - "What about the equations on the other cards?" Possible reasoning:
    - We can reason that all the other systems have one solution by a process of elimination—by noticing that they don't have the features of systems with many solutions or systems with no solutions.

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is for students to be able to recognize systems of equations that have one solution, infinitely many solutions, and no solutions. Students learned multiple ways to do this: by looking at the graphs of the two equations, by writing the equations in slope-intercept form and comparing them, and by noticing what happens when they attempt to solve the system algebraically. Students will consolidate this information by completing a graphic organizer during the debrief.

To help students summarize and organize the insights they gained in the lesson, have them complete (collaboratively in small groups or as a class) the graphic organizer found in their Student Workbook.

Students should start by labeling which row shows each of the following: one solution, infinitely many solutions, and no solutions. Discuss the characteristics of the graph and equations of each solution type with the class and have students use the space provided to record key takeaways.

Some important characteristics to ensure are addressed:

One Solution:

- The graphs of the equations in a system with one solution intersect at one point. The coordinates of the point are the one pair of values that are simultaneously true for both equations. When we solve the equations, we get exactly one solution.

Infinitely Many Solutions:

- The graphs of the equations in the second row appear to be the same line. This suggests that every point on the line is a solution to both equations, or that the system has infinitely many solutions.
- The two equations are equivalent equations; multiplying one of the equations by a factor will produce the other equation. The equations will have the same slope and vertical intercept. This can be seen by arranging the equations into slope-intercept form.
- When solved algebraically, the result is a true expression with no variables—usually $0 = 0$.

No Solutions:

- The graphs of the equations in the third row appear to be parallel. If the lines never intersect, then there is no common point that is a solution to both equations and the system has no solutions.
- The two equations can be multiplied by a factor to create the same coefficients of x and the same coefficients of y in each equation. However, this results in a different constant term in each equation. Since two equivalent expressions are equal to different values, there is no solution.
- When solved algebraically, the result is a false equation with no variables: for example, $0 = 10$.

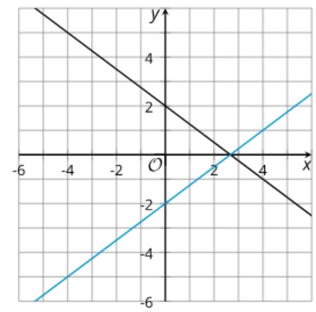
PLANNING NOTES

Student Lesson Summary and Glossary

We have seen many examples of a system where one pair of values satisfies both equations. Not all systems, however, have one solution. Some systems have many solutions, and others have no solutions.

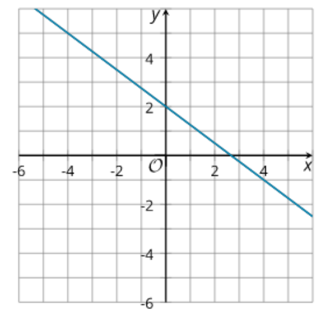
Let's look at three systems of equations and their graphs.

System 1: $\begin{cases} 3x+4y=8 \\ 3x-4y=8 \end{cases}$



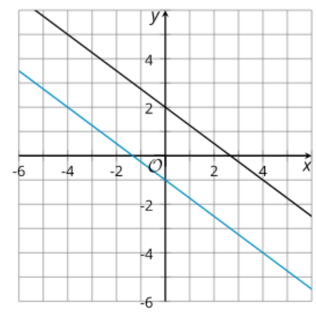
The graphs of the equations in system 1 intersect at one point. The coordinates of the point are the one pair of values that are simultaneously true for both equations. When we solve the equations, we get exactly one solution.

System 2: $\begin{cases} 3x+4y=8 \\ 6x+8y=16 \end{cases}$



The graphs of the equations in system 2 appear to be the same line. This suggests that every point on the line is a solution to both equations, or that the system has infinitely many solutions.

System 3: $\begin{cases} 3x+4y=8 \\ 3x+4y=-4 \end{cases}$



The graphs of the equations in system 3 appear to be parallel. If the lines never intersect, then there is no common point that is a solution to both equations and the system has no solutions.

How can we tell, without graphing, that system 2 indeed has many solutions?

- Notice that $3x+4y=8$ and $6x+8y=16$ are equivalent equations. Multiplying the first equation by 2 gives the second equation. Multiplying the second equation by $\frac{1}{2}$ gives the first equation. This means that any solution to the first equation is a solution to the second.

- Rearranging $3x+4y=8$ into slope-intercept form gives $y=\frac{8-3x}{4}$, or $y=2-\frac{3}{4}x$. Rearranging $6x+8y=16$ gives $y=\frac{16-6x}{8}$, which is also $y=2-\frac{3}{4}x$. Both lines have the same slope and the same y-value for the vertical intercept!

How can we tell, without graphing, that system 3 has no solutions?

- Notice that in one equation $3x+4y$ equals 8, but in the other equation it equals -4. Because it is impossible for the same expression to equal 8 and -4, there must not be a pair of x- and y-values that are simultaneously true for both equations. This tells us that the system has no solutions.

- Rearranging each equation into slope-intercept form gives $y=2-\frac{3}{4}x$ and $y=-1-\frac{3}{4}x$. The two graphs have the same slope but the y-values of their vertical intercepts are different. This tells us that the lines are parallel and will never cross.

Cool-down: No Graphs, No Problem (*5 minutes*)

Addressing: NC.M1.A-REI.6

Cool-down Guidance: More Chances
In optional Activity 3 in Lesson 18 (clue #4) and the Warm-up in Lesson 19 (graph b), systems of inequalities with no solutions are shown and will provide another opportunity to discuss when a system has no solutions and how to tell.

Graphing technology should not be used in this cool-down.

If time is limited, ask students to choose one system and explain how they could tell that it has no solutions or infinitely many solutions.

Cool-down

Mai is given these two systems of linear equations to solve.

System 1:

$$\begin{cases} 5x + y = 13 \\ 20x + 4y = 64 \end{cases}$$

System 2:

$$\begin{cases} 5x + y = 13 \\ 20x = 52 - 4y \end{cases}$$

She analyzed them for a moment, and then—without graphing the equations—said, "I got it! One of the systems has no solution and the other has infinitely many solutions!" Mai is right!

Which system has no solution, and which one has many solutions? Explain or show how you know (without graphing the equations).

Student Reflection:

What strategies did you add to your mathematical tool belt as we learned about systems of equations? Was there anything that you wish you had gotten to explore more during these systems lessons?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

As students worked together today, where did you see evidence of the mathematical community that is being established in the course?

Practice Problems

1. Here is a system of equations: $\begin{cases} 3x - y = 17 \\ x + 4y = 10 \end{cases}$

   a. Solve the system by graphing the equations (by hand or using technology).

   b. Explain how you could tell, without graphing, that there is only one solution to the system.

2. Consider this system of linear equations: $\begin{cases} y = \frac{4}{5}x - 3 \\ y = \frac{4}{5}x + 1 \end{cases}$

   a. Without graphing, determine how many solutions you would expect this system of equations to have. Explain your reasoning.

   b. Try solving the system of equations algebraically and describe the result that you get. Does it match your prediction?

3. How many solutions does this system of equations have? Explain how you know.

   $$\begin{cases} 9x - 3y = -6 \\ 5y = 15x + 10 \end{cases}$$

4. Select **all** systems of equations that have no solutions.

   a. $\begin{cases} y = 5 - 3x \\ y = -3x + 4 \end{cases}$

   b. $\begin{cases} y = 4x - 1 \\ 4y = 16x - 4 \end{cases}$

   c. $\begin{cases} 5x - 2y = 3 \\ 10x - 4y = 6 \end{cases}$

   d. $\begin{cases} 3x + 7y = 42 \\ 6x + 14y = 50 \end{cases}$

   e. $\begin{cases} y = 5 + 2x \\ y = 5x + 2 \end{cases}$

5. Here is a system of equations:

   $$\begin{cases} -x + 6y = 9 \\ x + 6y = -3 \end{cases}$$

   Would you rather use subtraction or addition to solve the system? Explain your reasoning.

   (From Unit 3, Lesson 10)

6. Solve each system of equations without graphing.

a. $\begin{cases} 2v + 6w = -36 \\ 5v + 2w = 1 \end{cases}$

b. $\begin{cases} 6t - 9u = 10 \\ 2t + 3u = 4 \end{cases}$

(From Unit 3, Lesson 11)

7. Here is a system of linear equations:

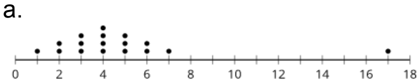
$$\begin{cases} 6x - y = 18 \\ 4x + 2y = 26 \end{cases}$$

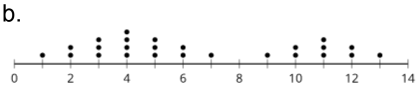
Select **all** the steps that would help to eliminate a variable and enable solving.

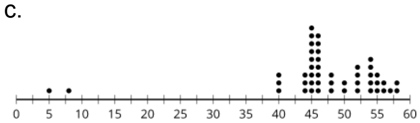
a. Multiply the first equation by 2, then subtract the second equation from the result.
b. Multiply the first equation by 4 and the second equation by 6, then subtract the resulting equations.
c. Multiply the first equation by 2, then add the result to the second equation.
d. Divide the second equation by 2, then add the result to the first equation.
e. Multiply the second equation by 6, then subtract the result from the first equation.

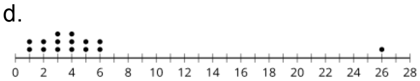
(From Unit 3, Lesson 11)

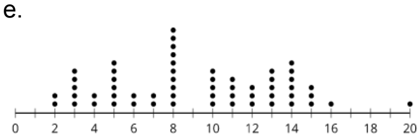
8. Select **all** the dot plots that appear to contain outliers.

a.


b.


c.


d.


e.


(From Unit 1)

9. Lin was looking at the equation $2x - 32 + 4(3x - 2462) = 14x$. She said, "I can tell right away there are no solutions, because on the left side, you will have $2x + 12x$ and a bunch of constants, but you have just $14x$ on the right side." Do you agree with Lin? Explain your reasoning.[2]

(Addressing NC.8.EE.8)

10. Han was looking at the equation $6x - 4 + 2(5x + 2) = 16x$. He said, "I can tell right away there are no solutions, because on the left side, you will have $6x + 10x$ and a bunch of constants, but you have just $16x$ on the right side." Do you agree with Han? Explain your reasoning.[3]

(Addressing NC.8.EE.8)

[2] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).
[3] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html (see above).

Lessons 13 & 14: Checkpoint

PREPARATION

Lesson Goals	Learning Targets
• Find the midpoint of a line, given two endpoints as ordered pairs. • Determine the second endpoint of a line, given the midpoint and other endpoint.	• I can find the midpoint between two points on a coordinate graph. • I can find an endpoint of a line if I am given the midpoint and the other endpoint.

Lesson Narrative

This is a Checkpoint day. These lessons have three main purposes: 1. differentiated and small-group instruction; 2. opportunities for students to participate in various learning stations to refine and extend previous learning; and 3. the opportunity for students to complete the next unit's Check Your Readiness (CYR). Administering the CYR at this point in the unit allows plenty of time for the data to inform the next unit's instruction. Checkpoint days consist of two lessons (one full block) and are structured as four 20-minute stations that students rotate between. There are a total of eight stations students can engage with. Since students will not be able to participate in all eight stations, please note that Station A (Unit 4 Check Your Readiness) and Station B (In The Middle) are required for all students.

A. Unit 4 Check Your Readiness (*Required*)
B. What's the (Mid)point? (*Required*)
C. Teacher-led Small-group Instruction
D. One, No, Infinitely Many
E. Systems in Context
F. Parallel and Perpendicular Lines
G. Micro-Modeling
H. Are You Ready For More?

How will you determine which stations individual students participate in? How can you organize the stations in a way that empowers students' development of mathematical identities, provides essential support, and limits the appearance of "smart" or "not-smart" assignments?

Focus and Coherence

Addressing
NC.M1.G-GPE.6: Use coordinates to find the midpoint or endpoint of a line segment.

Agenda, Materials, and Preparation

- **Station A** (*Required, 20 minutes*)
  - Unit 4 Check Your Readiness (print 1 copy per student)
- **Station B** (*Required, 20 minutes*)
  - Assign Desmos activity to class or generate a single session code
- **Station C** (*20 minutes*)
- **Station D** (*20 minutes*)
- **Station E** (*20 minutes*)
- **Station F** (*20 minutes*)
- **Station G** (*20 minutes*)
- **Station H** (*20 minutes*)
  - Are You Ready For More? tasks in Student Workbook from past lessons (or optional: print 1 blackline master per student)

STATIONS

Station A: Unit 4 Check Your Readiness (*Required, 20 minutes*)

Remind students that it is really important that their responses to these questions accurately represent what they know. Ask them to answer what they can to the best of their ability. If they get stuck, they should name what they don't know or understand.

Station B: What's the (Mid)point? (*Required, 20 minutes*)

Addressing: NC.M1.G-GPE.6

Station B is a required station that introduces the concept of midpoint. Students will use their prior knowledge to create their own working definition of midpoint and learn how to calculate the midpoint of two points on a graph. In previous grades, students learned that the mean is the average of the numbers and is the balance point for those numbers. In this lesson, they will recall that information and build upon it to craft a method for finding the midpoint. Further opportunities for calculating midpoint are provided in the practice problems of subsequent lessons.

Step 1

- Prior to class, go to https://bit.ly/CMSMidpoint to access the Activity Builder for this Desmos station.
- Give students access to this station by clicking on "Assign" and choose either "Assign to Your Classes" or "Single Session Code."
  - A class must be created, and students added to it, in a teacher's Desmos account in order to use the "Assign to Your Classes" option. Students will see a "Start" button next to the activity title when logged in on the student.desmos.com page.
  - In order to do this activity without creating a class in Desmos, a "Single Session Code" can be generated to give to students. Instruct students to go to student.desmos.com and enter the single session code.

Step 2

- Student progress can be monitored by clicking "View Dashboard" underneath Activity Sessions on the Desmos Activity Builder page. From this dashboard, student pacing can be adjusted, the activity can be paused for students, and student names can be anonymized.
- Provide feedback to individual students by clicking the chat icon at the top of the student work window of a particular slide.

Station B

Follow your teacher's directions to access this station's Desmos activity. Use the available space below to show your work.

Station C: Teacher-led Small-group Instruction (*20 minutes*)

Use student cool-down data, Check Your Readiness Unit 3 data, and informal formative assessment data from Unit 3 (Lessons 1–12) to provide targeted small-group instruction to students who demonstrate the need for further support or challenge on topics taught up to this point.

Potential topics:

- Equations and their graphs
- Calculating and analyzing slope
- Solving systems of equations
- Parallel and perpendicular lines
- Calculating perimeter and area of triangles and quadrilaterals in the coordinate plane

Station D: One, No, Infinitely Many (*20 minutes*)

Addressing: NC.M1.A-REI.6

Station D gives students another opportunity to apply what they learned about the features of systems of linear equations with one solution, no solution, and infinitely many solutions. In earlier lessons, students were given systems of equations and were asked to determine the number of solutions. Here, they are given one equation and are asked to write a second one such that the two equations form a system with one solution, no solution, and many solutions.

To answer part a (a system with one solution), students could write a second equation with randomly chosen parameters. Answering parts b and c, however, relies on an understanding of what "no solution" and "infinitely many solutions" mean and how these conditions are visible in the pair of equations and in the graph.

For example, students could reason that in a system with no solution:

- The two equations have the same variable expressions on one side but different numbers on the other side, and then write a second equation accordingly. For instance, if $5x - 2y$ is equal to 10, it cannot also be equal to 4, so $5x - 2y = 10$ and $5x - 2y = 4$ would form a system with no solution.

- The graphs of the equations have the same slope, but they cross the vertical axis at different points. Rewriting $5x - 2y = 10$ in the form of $y = mx + b$ gives $y = \frac{5}{2}x + 10$. A second equation with a coefficient of $\frac{5}{2}$ for x but a different constant would have a graph that is parallel to the graph of the first equation, forming a system with no solution.

Regardless of the form students use to write the second equation, they need to choose the parameters strategically to achieve a system with the desired number of solutions. The work here prompts students to look for and make use of structure (MP7).

Station D

Find a partner. Solve question 1 a-c independently. When completed, compare your responses with your partner and answer part d together. Repeat for questions 2 and 3.

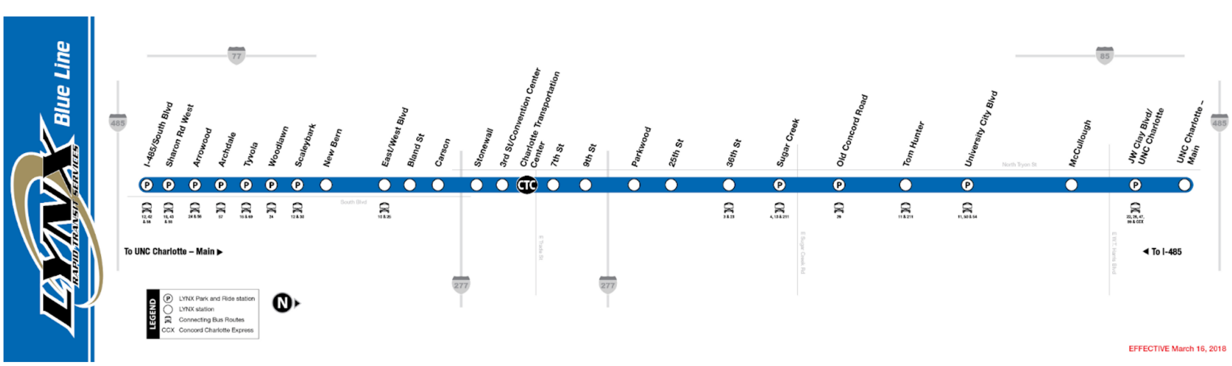
	a. Create a second equation that would make a system of equations with one solution:	b. Create a second equation that would make a system of equations with no solution:	c. Create a second equation that would make a system of equations with infinitely many solutions:	d. Compare your equations with your partner. Did you create the same equations? If not, what was different? What was similar? Were you both correct? How do you know?
1. $y = -3x - 12$				
2. $5x - 2y = 10$				
3. $-4x - 2y = 10$				

DO THE MATH

PLANNING NOTES

Station E: Systems in Context (*20 minutes*)

Station E



	Adult	Seniors 62+	ADA-disabled	Student K-12
One-way tickets	$2.20	$1.10	$1.10	$1.10
Weekly unlimited pass	$30.80	$30.80	$30.80	$30.80
Monthly unlimited pass	$88.00	$44.00	$44.00	$88.00
10-Ride	$22.00	$9.35	$9.35	$22.00

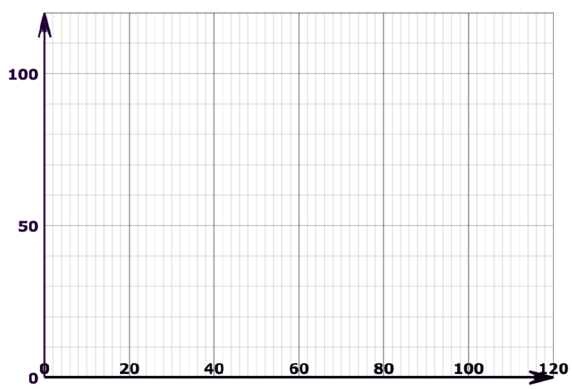
1. It costs $88.00 for a monthly unlimited pass to ride the Light Rail. Without the pass, it costs $2.20 per ride for adults and $1.10 per ride for students.

    a. How much does it cost an adult to ride the Light Rail one time? Five times? Twenty times? Fifty times? Try at least three more values.

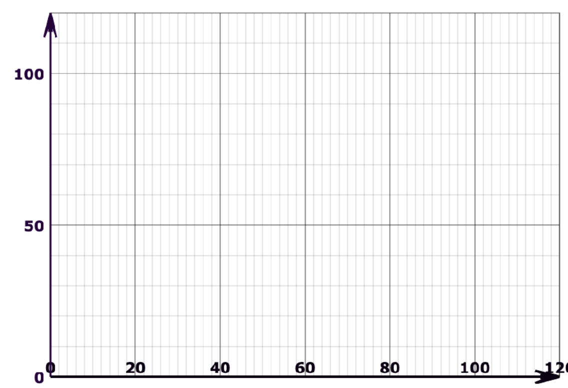
    b. How much does it cost a student to ride the Light Rail one time? Five times? Twenty times? Fifty times? Try at least three more values.

    c. Construct a graph to compare the costs of the monthly pass and individual one-way tickets for adults and a graph to compare the costs of the monthly pass and individual one-way tickets for students.

Monthly pass



Individual one-way tickets

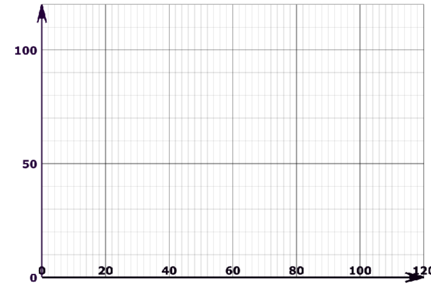


    d. Determine how many times an adult would have to ride the light rail before buying the monthly pass is a better deal.

    e. Determine how many times a student would have to ride the light rail before buying the monthly pass is a better deal.

    f. Write a description of how you might convince someone whether they should or should not buy a monthly pass.

2. A family came to visit Charlotte on vacation and since they are not staying long, they choose to only buy one-way tickets. The family (which consists of only adults and children in grades K–12) bought 15 one-way tickets that cost them a total of $23.10. Write a system of equations and solve to determine how many adult tickets and how many student tickets they bought.



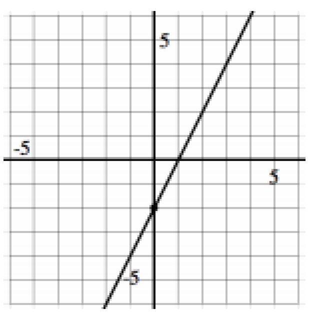
DO THE MATH

PLANNING NOTES

Station F: Parallel and Perpendicular Lines[1] (*20 minutes*)

Station F

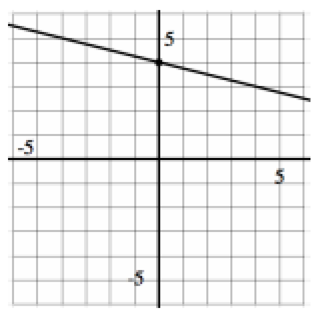
1. Graph a line *parallel* to the given line.



Equation for given line:

Equation for new line:

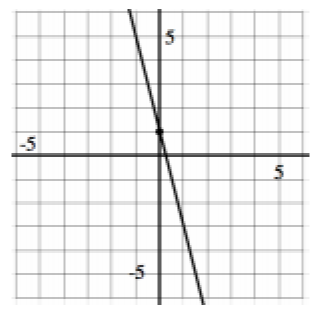
2. Graph a line *parallel* to the given line.



Equation for given line:

Equation for new line:

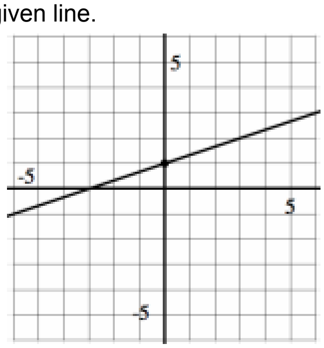
3. Graph a line *parallel* to the given line.



Equation for given line:

Equation for new line:

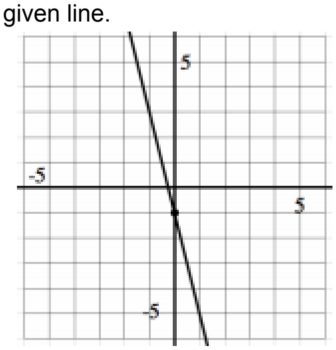
4. Graph a line *perpendicular* to the given line.



Equation for given line:

Equation for new line:

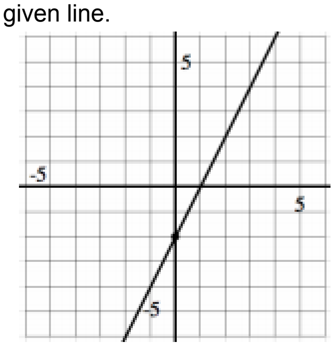
5. Graph a line *perpendicular* to the given line.



Equation for given line:

Equation for new line:

6. Graph a line *perpendicular* to the given line.



Equation for given line:

Equation for new line:

[1] Adapted from Math 1, Module 1, Lesson 1.2 Mathematics Vision project http://www.mathematicsvisionproject.org, licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0)

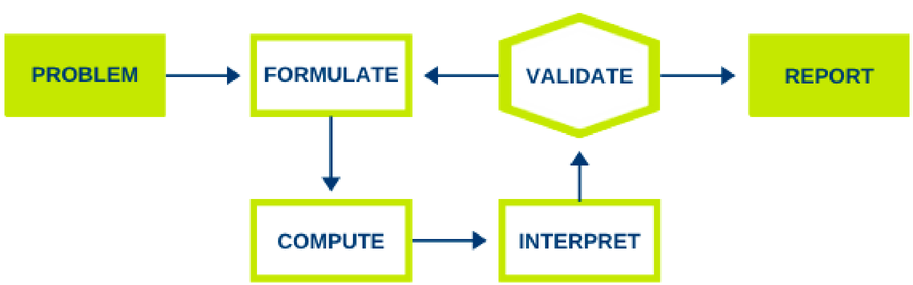
DO THE MATH	PLANNING NOTES

Station G: Micro-Modeling (*20 minutes*)

Instructional Routine: Aspects of Mathematical Modeling

Modeling is the link between the mathematics students learn in school and the problems they will face in college, career, and life. Time spent on modeling in Math 1 is crucial, as it prepares students to use math to handle technical subjects in their further studies, and problem solve and make decisions that adults regularly encounter in their lives.

ASPECTS OF MATHEMATICAL MODELING	***What Is This Routine?*** In activities tagged with this routine, students engage in scaled-back modeling scenarios, for which students only need to engage in a part of a full modeling cycle. For example, they may be selecting quantities of interest in a situation or choosing a model from a list. ***Why This Routine?*** Mathematical modeling is often new territory for both students and teachers. Activities tagged as *Aspects of Mathematical Modeling* offer opportunities to develop discrete skills in the supported environment of a classroom lesson to make success more likely when students engage in more open-ended modeling.



These tasks can also be offered as additional practice problems at any point in the unit or used in the teacher-led small-group instruction.

Station G[2]

Select one of the following questions. Spend about 10 minutes writing your response. Leave your draft response on the table when it is ready for feedback, and pick up another student's draft to review and provide feedback. Note any parts of the writing that are clear, any parts that are confusing, and any parts that seem unfinished. Give feedback on sticky notes or use different colored sticky arrows so that the original student's work doesn't get messed up. After someone has given your draft feedback, use the remaining time to improve your response.

2 Adapted from Achievethecore.org

1. The following question was posted on an internet gardening forum:

   I am in charge of purchasing soil for my neighborhood's community garden and I am trying to figure out how many yards of soil I need but have no idea how much a yard of soil actually is. The nursery says they deliver 6 yards in a dump truck. I realize that a yard is 3 feet. But what is a yard of soil—is that 3 feet long and 3 feet high? I'm clueless! LOL

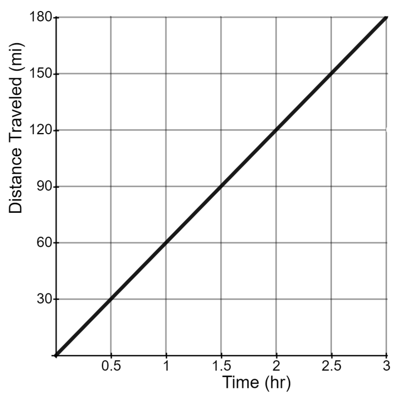
   Write a helpful response to the person who posted this question.

2. Each graph below shows the relationship between distance traveled and time for a different train (Train A, Train B, and Train C).

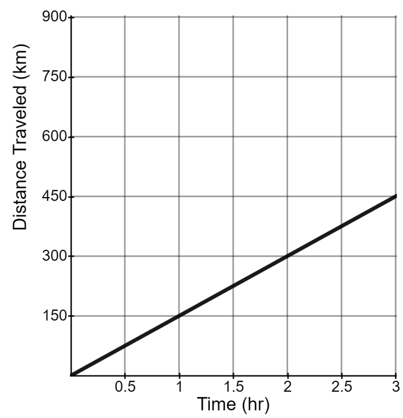
   Which train was traveling fastest during the interval of time shown? Justify your answer with a thorough explanation using words, numbers, and/or visuals.

   Distance Traveled Vs. Time

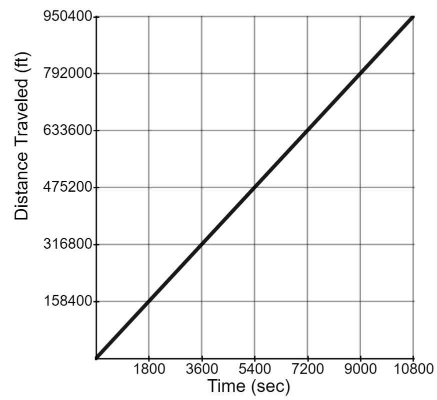
Train A



Train B



Train C



DO THE MATH

PLANNING NOTES

Station H: Are You Ready For More? (*20 minutes*)

Students who did not complete the "Are You Ready For More?" task statements from Lessons 1, 4, 6, 7, 8, 9, and 10 can do so in Station H. This is a great opportunity for students to expand their thinking. These tasks can also be offered as additional practice problems or used in the teacher-led small-group instruction.

Station H

1. A 450-gallon tank full of water is draining at a rate of 20 gallons per minute.

   a.
      - How many gallons will be in the tank after 7 minutes?
      - How long will it take for the tank to have 200 gallons?
      - Write an equation that represents the relationship between the gallons of water in the tank and minutes the tank has been draining.
      - Graph your equation using graphing technology. Mark the points on the graph that represent the gallons after 7 minutes and the time when the tank has 200 gallons. Write down the coordinates.
      - How long will it take until the tank is empty?

   b. Write an equation that represents the relationship between the gallons of water in the tank and *hours* the tank has been draining.

   c. Write an equation that represents the relationship between the gallons of water in the tank and *seconds* the tank has been draining.

   d. Graph each of your new equations. In what way are all of the graphs the same? In what way are they all different?

   e. How would these graphs change if we used quarts of water instead of gallons? What would stay the same?

   (From Unit 3, Lesson 1)

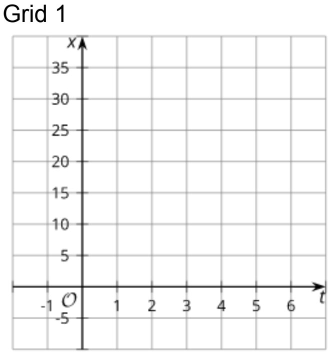
2. Another way to describe a line, or other graphs, is to think about the coordinates as changing over time. This is especially helpful if we're thinking about tracing an object's movement. This example describes the x- and y-coordinates separately, each in terms of time, t.

   a. On the first grid, create a graph of $x = 2 + 5t$ for $-2 \le t \le 7$ with x on the vertical axis and t on the horizontal axis.

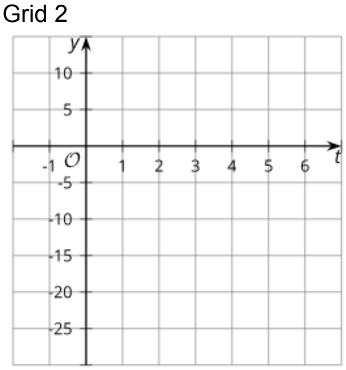
   b. On the second grid, create a graph of $y = 3 – 4t$ for $-2 \le t \le 7$ with y on the vertical axis and t on the horizontal axis.

   c. On the third grid, create a graph of the set of points $(2 + 5t, 3 – 4t)$ for $-2 \le t \le 7$ on the xy-plane.

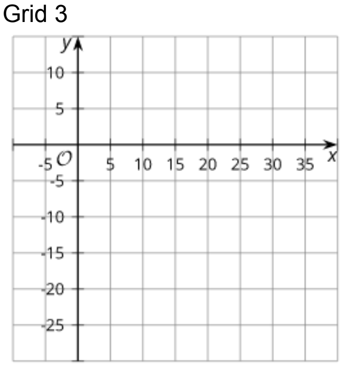
Grid 1



Grid 2



Grid 3



(From Unit 3, Lesson 4)

3.

a. Line l is represented by the equation $y = \frac{2}{3}x + 3$. Write an equation of the line perpendicular to l, passing through $(-6, 4)$. Call this line p.

b. Write an equation of the line perpendicular to p, passing through $(3, -2)$. Call this line n.

c. What do you notice about lines l and n? Does this always happen? Show or explain your work.

(From Unit 3, Lesson 6)

4.

a. Make up equations for two lines that intersect at $(4, 1)$.

b. Make up equations for three lines whose intersection points form a triangle with vertices at $(-4, 0)$, $(2, 9)$, and $(6, 5)$.

(From Unit 3, Lesson 8)

5. Solve this system with four equations. $\begin{cases} 3x + 2y - z + 5w = 20 \\ y = 2z - 3w \\ z = w + 1 \\ 2w = 8 \end{cases}$

(From Unit 3, Lesson 9)

6. This system has three equations: $\begin{cases} 3x + 2y - z = 7 \\ -3x + y + 2z = -14 \\ 3x + y - z = 10 \end{cases}$

a. Add the first two equations to get a new equation.

b. Add the second two equations to get a new equation.

c. Solve the system of your two new equations.

d. What is the solution to the original system of equations?

(From Unit 3, Lesson 10)

DO THE MATH

PLANNING NOTES

TEACHER REFLECTION

In what ways did the students surprise you over the past two lessons? What were the good surprises? What surprises can you learn from?

Reflect on the way you chose to group the students for this lesson (i.e., your method for grouping). What group choices worked well to increase student learning and what will you do differently when grouping next time?

Lesson 15: Graphing Linear Inequalities in Two Variables (Part One)

PREPARATION

Lesson Goals	Learning Targets
• Find the solution to a two-variable inequality by graphing a related two-variable equation and determining the correct region for the solution. • Interpret, in context, points on the graphs of equations and in the solution region of inequalities in two variables. • Write inequalities in two variables to represent the constraints in a situation and identify possible solutions by reasoning.	• I can find the solutions to a two-variable inequality by using the graph of a related two-variable equation. • Given a two-variable inequality that represents a situation, I can interpret points in the coordinate plane and decide if they are solutions to the inequality. • I can write inequalities to describe the constraints in a situation.

Lesson Narrative

In this lesson, students explore how to determine solutions to two-variable linear inequalities by studying them in context. They reason through solutions that satisfy the described constraints and then write inequalities in two variables to represent the constraints. Additionally, students interpret the points on a boundary line and on either side of it in terms of the situation.

The work here illustrates that the solution region represents the set of values that satisfy the constraint in a situation (MP2). Interpreting the solutions contextually also engages students in an aspect of mathematical modeling (MP4). It enables students to see that, while some values might make an inequality true, they might not be feasible or appropriate in the situation. The sequence of activities starting with Warm-up and moving into Activity 1 provides opportunities to make generalizations based on repeated reasoning (MP8), since students first find numbers that meet a constraint and then use variables in place of those numbers to write an equation and an inequality.

Because reasoning about the solution region of an inequality is important here, graphing technology should not be used. Students will have opportunities to use graphing technology to solve inequalities in two variables in upcoming lessons.

What strategies or representations do you anticipate students might use in this lesson?

Focus and Coherence

Building On	Addressing	Building Towards
NC.6.EE.2: Write, read, and evaluate algebraic expressions. • Write expressions that record operations with numbers and with letters standing for numbers. • Identify parts of an expression using mathematical terms and view one or more of those parts as a single entity. • Evaluate expressions at specific values of their variables using expressions that arise from formulas used in real-world problems. **NC.M1.A-REI.10:** Understand that the graph of a two variable equation represents the set of all solutions to the equation.	**NC.M1.A-REI.12:** Represent the solutions of a linear inequality or a system of linear inequalities graphically as a region of the plane.	**NC.M1.A-CED.3:** Create systems of linear equations and inequalities to model situations in context.

Agenda, Materials, and Preparation

- **Bridge** *(Optional, 5 minutes)*
- **Warm-up** (*10 minutes*)
- **Activity 1** (*10 minutes*)
- **Activity 2** *(10 minutes)*
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U3.L15 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.6.EE.2

The purpose of this bridge is to illustrate how different, but equivalent, algebraic expressions can reveal different information about a situation represented by those expressions. Working with equivalent expressions is an important skill for solving linear equations and interpreting them in contexts. If possible, display part a for students to answer before they explore parts b and c in their student workbooks.

Student Task Statement

Mai is at an amusement park. She bought 14 tickets, and each ride requires 2 tickets.[1]

a. Write an expression that gives the number of tickets Mai has left in terms of x, the number of rides she has already gone on. Find at least one other expression that is equivalent to it.

b. $14 - 2x$ represents the number of tickets Mai has left after she has gone on x rides. How can each of the following numbers and expressions be interpreted in terms of tickets and rides?
- 14
- -2
- $2x$

c. $2(7 - x)$ also represents the number of tickets Mai has left after she has gone on x rides. How can each of the following numbers and expressions be interpreted in terms of tickets and rides?
- 7
- $(7 - x)$
- 2

[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

DO THE MATH

PLANNING NOTES

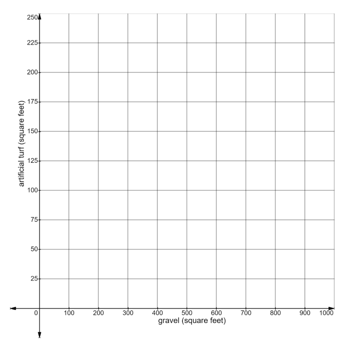
Warm-up: Landscaping Options (*10 minutes*)

Instructional Routine: Three Reads (MLR6)	
Addressing: NC.M1.A-REI.10	**Building Towards:** NC.M1.A-CED.3

In this lesson, students will be writing linear inequalities that represent constraints in situations and graphing the solution regions. To prepare for that work, students review writing and graphing an equation that represents a situation and interpret points on the graph of the line.

Step 1

- Use the *Three Reads* routine to support comprehension of this word problem.
  - First Read: Without displaying the task, read the context aloud to the class: "A homeowner is making plans to landscape her yard to make it more low-maintenance. She plans to hire professionals to install artificial turf in some parts of the yard and gravel in other parts. Artificial turf costs $15 per square foot to install and gravel costs $3 per square foot to install. She may use a combination of the two materials in different parts of the yard. Her budget is $3,000."



  - Ask students: "What is this situation about? What is going on here?"
    - Let students know the focus is just on the situation, not on the numbers (for example, students might say "it's about low-maintenance landscaping materials" or "it's about the cost of turf and gravel").
    - Spend less than a minute scribing their understanding of the situation in a place where all can see. Do not correct students, but do clarify any unfamiliar words.
    - Display the following images of artificial turf and gravel for all to see. Share with students that these are examples of low-maintenance landscaping materials. If needed, explain or show additional images of artificial turf and gravel to students who might be unfamiliar with these landscaping terms.

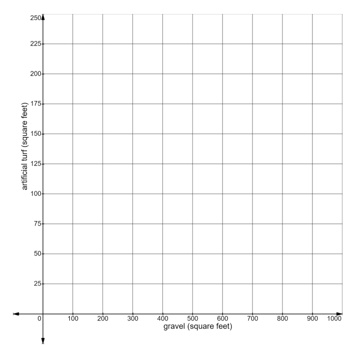
- Second Read: Display the task context without the problems, and ask a student volunteer to read it aloud to the class again.
    - Ask: “What are the quantities in this situation? A quantity is something that can be counted or measured.”
    - Again, spend less than a minute scribing student responses.
    - Encourage students to identify quantities that are named in the problem explicitly, and any quantities that may be implicit. For each quantity (for example, “3,000”), ask students to add details (for example, “the total amount she can spend is $3,000”).
- Third Read: Ask students to brainstorm possible strategies to answer the question, “What combinations of turf and gravel would total $3,000?”
    - Ask: “How might we approach this question? What is the first thing you will do?”
    - Spend 1–2 minutes scribing student ideas as they brainstorm possible starting points.
    - Be sure to stop any students who begin to share a complete solution; the goal is to crowdsource some starting points.

Student Task Statement

A homeowner is making plans to landscape her yard to make it more low-maintenance. She plans to hire professionals to install artificial turf in some parts of the yard and gravel in other parts.

Artificial turf costs $15 per square foot to install and gravel costs $3 per square foot to install. She may use a combination of the two materials in different parts of the yard. Her budget is $3,000.

1. Write an equation that represents the square feet of gravel, x, and the square feet of artificial turf, y, that she could afford if she used her entire budget.
2. On the coordinate plane, sketch a graph that represents your equation. Be prepared to explain your reasoning.
3. What does the point (500,100) mean?
4. What do the solutions to the equation of the line you graphed mean?



Step 2

Invite students to share their equation and graph. Discuss with students:

- "In this situation, what does a point on the line mean?" (a combination of square feet of artificial turf and square feet of gravel that the homeowner could have if she spent her entire budget)
- "What does the vertical intercept of the graph mean?" (the square feet of artificial turf she could have if she installs no gravel)
- "What does the horizontal intercept of the graph tell us?" (the square feet of gravel she could install if she installs no artificial turf)

DO THE MATH

PLANNING NOTES

Activity 1: Rethinking Landscaping (*10 minutes*)

Instructional Routine: Aspects of Mathematical Modeling
Addressing: NC.M1.A-REI.12

This is the first of a series of activities in which students write an inequality to represent a constraint in a situation. Students interpret the coordinate pairs of points on either side of the graph of the line created in the warm-up and test the pairs of values to see if they also satisfy the constraints of the problem. They then use these observations to speculate that the solution to a two-variable inequality is a region in the coordinate plane.

Students also engage in *Aspects of Mathematical Modeling* as they consider whether all the points in the solution region are necessarily meaningful or feasible in the situation. In doing so, they reason quantitatively and abstractly (MP2) and practice evaluating the reasonableness of their solutions in context (MP4).

Reminder: graphing technology should not be used in this activity and the other activities in the lesson so students can focus on representing the structure of an inequality with a given context.

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Give them quiet time to work on the first two problems, followed by time to share their thinking before moving on to the rest of the activity.

Monitoring Tip: As students share their thinking for problems 1 and 2, listen for students that can articulate why one point satisfies the constraint, but the other does not. Plan to have these students share out to the whole class in Step 2.

RESPONSIVE STRATEGY

Chunk this task into manageable parts for students who benefit from support with organizational skills in problem solving. Check in with students after the first 2–3 minutes of work time. Invite 1–2 students to share their strategies for exploring solution sets of inequalities.

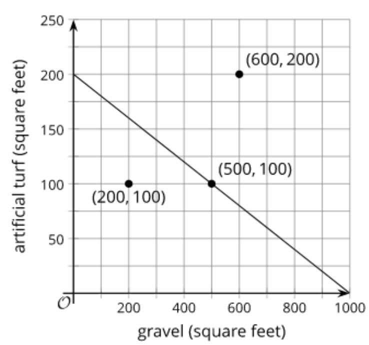
Supports accessibility for: Organization; Visual-spatial processing

Student Task Statement

In the previous problem, we encountered a homeowner looking to update her landscape to include low-maintenance materials.

She is considering artificial turf, which costs $15 per square foot to install, and gravel, which costs $3 per square foot. She may use a combination of the two materials in different parts of the yard. Her budget is still $3,000.

Here is the graph representing some of the constraints in this situation once again.



1. The point $(600, 200)$ is located to the right and above the line.
   a. Does that combination of turf and gravel meet the homeowner's constraints? Explain or show your reasoning.
   b. Choose another point in the same region (to the right and above the line). Check if the combination meets the homeowner's constraints.
   c. What do the points above the line represent?
2. The point $(200, 100)$ is located to the left and below the line.
   a. Does that combination of turf and gravel meet the homeowner's constraints? Explain or show your reasoning.
   b. Choose another point in the same region (to the left and below the line). Check if the combination meets the homeowner's constraints.
   c. What do the points below the line represent?
3. Write an inequality that represents the constraints in this situation. Explain what the solutions to this inequality would mean.

Step 2

- Select students to share their interpretations of the two points on either side of the line. Make sure students understand that one point represents a combination of gravel and artificial turf that meets the budget constraint, and that the other point does not. The region in which each point belongs can be interpreted in the same way.
- Make sure students understand why points on the boundary line are included in the solution set of $3x + 15y \le 3{,}000$.
- Facilitate a discussion to encourage students to evaluate the reasonableness of their solutions in terms of the situation being modeled by asking questions such as:
  - "Both $(100, 50)$ and $(50, 100)$ satisfy the inequality. Both points mean a total of 150 square feet of landscape materials. Does it make a difference which option is chosen?" (It does not make a difference in terms of the total area, but it does in terms of cost. It might also make a difference to the plants and to the overall appearance of the yard.)
  - "All the points below the line represent amounts of gravel and turf that are within the homeowner's budget. Are all these options equally good and desirable?" (Most likely not. For example, if the homeowner needs both materials, pairs such as $(0, 100)$ or $(450, 0)$ are probably not desirable because they mean buying only one material but not the other. The point $(0, 0)$ is also below the line, but buying no materials is probably also not an option because it would not help the homeowner with her landscaping goals.)
- Once students are convinced that the points below the line form the solution set to the inequality, shade those points and tell students that we can call the solution set to a two-variable inequality the solution region.

DO THE MATH

PLANNING NOTES

Activity 2: Charity Concerts (*10 minutes*)

Addressing: NC.M1.A-REI.12	**Building Towards:** NC.M1.A-CED.3

This activity offers an additional opportunity for writing a linear inequality in two variables to represent constraints and for graphing and interpreting the solutions.

In the previous scenario, the solution region of the inequality $Ax + By \le C$ lies below the boundary line. Some students might misinterpret this to imply that the solutions will always be below the line for inequalities of the form $Ax + By \le C$ and above the line for inequalities of the form $Ax + By \ge C$. In this activity, students encounter an example in which the symbol $\ge$ does not correspond to a solution region above the boundary line. (In the given situation, pairs of values that generate more revenue for the concert are points below the graph of $Ax + By = C$.)

The work here reinforces the importance of reasoning about points on either side of a boundary line, rather than simply assuming that < or ≤ means shading below the line and > or ≥ means shading above the line. It allows students to practice reasoning quantitatively and abstractly (MP2). Students will have more practice testing points and reasoning about solutions in the next lesson.

RESPONSIVE STRATEGY

Provide appropriate reading accommodations and supports to ensure student access to written directions, word problems and other text-based content. Use this opportunity to show students an example of reading for understanding. Demonstrate pausing after each sentence, and rereading to stress important information.

Supports accessibility for: Language; Conceptual processing

Step 1

- Give students 2 minutes of quiet think time to read the scenario, write an equation, and graph it.
- Next, invite students to share their equations and graphs with a partner and resolve any differences.
- Once student pairs are confident that their work is correct, ask students to continue working independently on the remaining questions.

Monitoring Tip: As students work, pay attention to how students determine how to shade the appropriate region, and which ordered pairs satisfy their inequality. Be prepared to have students who tested specific pairs of values share their work.

Advancing Student Thinking: Some students may find it challenging to graph the boundary line ($25t - 1250c = 20,000$) by identifying the intercepts. The horizontal intercept is fairly easy to find, but the graph intersects the vertical axis at a negative value (and a negative number of concerts does not make sense in this situation). Ask students to find at least one other point (besides the vertical intercept) that is a solution to the equation.

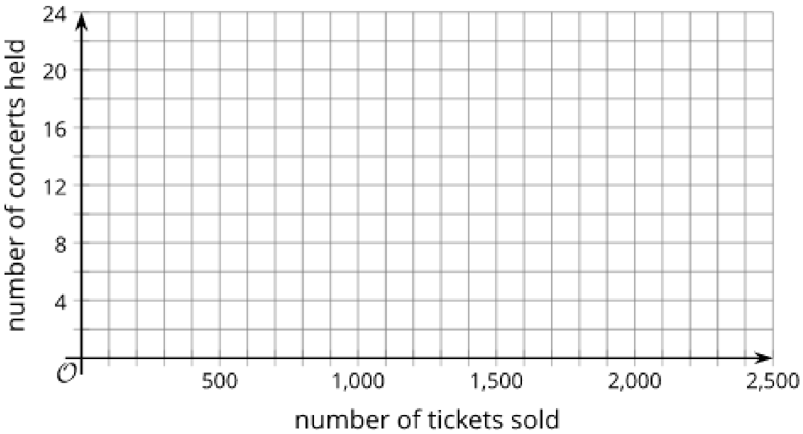
Students who rewrite the equation in slope-intercept form and find the slope to be 0.02 may also find it difficult to interpret. Ask them to try writing the slope as a fraction $\left(\frac{2}{100}\right)$.

Student Task Statement

A popular band is trying to raise at least $20,000 for charity by holding multiple concerts at a park. It plans to sell tickets at $25 each. For each two-hour concert, the band would need to pay the park $1,250 in fees for security, cleaning, and traffic services.

The band needs to find the combinations of total number of tickets sold, t, and number of concerts held, c, that would allow it to reach its fundraising goal.

1. Write an equation to represent raising **exactly** $20,000 for charity.
2. Graph the line containing the solutions to your equation on the coordinate plane.



3. Name two possible combinations of number of tickets sold and number of concerts held that would allow the band to meet its goal. Plot these ordered pairs on the graph above.

4. Which combination of tickets and concerts would mean **more** money for charity:

   a. 1,300 tickets and 10 concerts, or 1,300 tickets and 5 concerts?
   b. 1,600 tickets and 16 concerts, or 1,200 tickets and 9 concerts?
   c. 2,000 tickets and 4 concerts, or 2,500 tickets and 10 concerts?

5. Write the inequality that represents raising **at least** $20,000 for charity.

6. Using the combinations of tickets and concerts presented in question 4, plot the ordered pairs that would satisfy the inequality in question 5 on the above graph.

Step 2

- Invite students to share their inequality and their graph.
- Facilitate a discussion focused on questions 3 and 4 around how students knew which combinations of tickets and concerts would enable the band to meet its goal and would raise more money. Highlight responses that involve testing pairs of values to see if they satisfy the equation or to make comparisons.
- If not mentioned in students' explanations, point out that even though the inequality has a ≥ symbol, the solutions to the inequality were below the boundary line, not above it. Encourage students to hypothesize why this is the case in this situation.

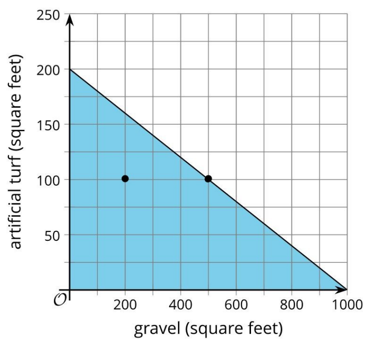
DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

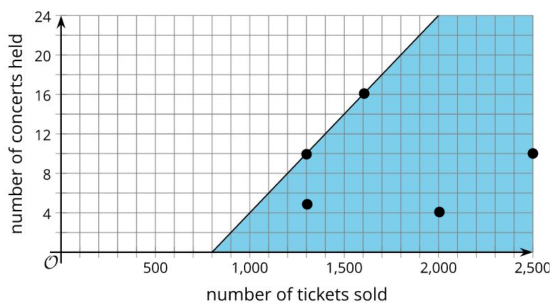
In this lesson, students reason about the solution region to a two-variable inequality.

Display the inequalities and graphs from both activities in the lesson. Share with students that the ordered pairs that satisfy the inequalities always fall within a defined region. Ask students to compare the regions containing the solutions and think about what they tell us about the constraints of the situation they represent.

$3x + 15y \leq 3,000$



$25t - 1,250c \geq 20,000$



Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion. Discuss questions such as:

- "How are the solution regions of the two inequalities alike?" (Each region represents all pairs of values that meet a money-related constraint in a situation. They cover a part of the plane. They stop at a line.)
- "How do we know where the boundary line would be for each graph?" (It is the graph of a related equation.)
- "For each inequality, how did we find out which side of the line contains the solutions?" (We tested one or more pairs of values on each side and saw if—when substituted for the variables—they made the inequality true.)
- "In the first situation, some pairs of values that are in the solution region don't make sense in the situation. Can you explain why a pair such as $(800, 1)$, which is in the solution region, might not be a reasonable option for the homeowner?" (It doesn't quite make sense to cover 1 square foot of the garden with artificial turf and the rest with gravel. The area of the garden might be a lot greater or a lot less than 801 square feet.)
- "In the second situation, we know that fractional values are not meaningful even though they are in the shaded region. Can you think of other reasons that some points in the solution region might not make sense?" (A point like $(2000, 20)$ would be in the solution region, but the venue might not be available for that many concerts)

Some students may point out that it is possible to reason about the side that contains the solutions by reasoning about each context. In these particular examples, this can be done intuitively and correctly. When the boundary line represents the cost of two quantities exactly on budget, smaller values of each quantity would lead to costs that are below the budget. When the boundary line represents a profit of $100 from selling two kinds of products, a greater number of each product would mean a greater profit.

Emphasize, however, that it is not always the case that the solution region could be reasoned easily or correctly from the context, so it is always a good idea to verify using another method. Activity 2 illustrates this point.

PLANNING NOTES

Student Lesson Summary and Glossary

Inequalities in two variables can represent constraints in real-life situations. Graphing their solutions can enable us to solve problems.

Suppose a café is purchasing coffee and tea from a supplier and can spend up to $\$1,000$. Coffee beans cost $\$12$ per kilogram and tea leaves cost $\$8$ per kilogram.

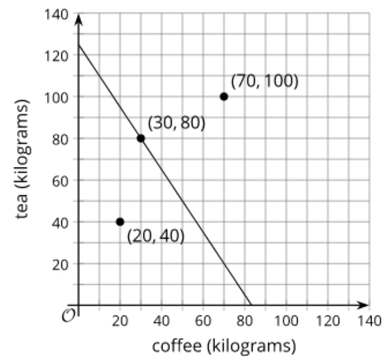
Buying c pounds of coffee beans and t pounds of tea leaves will therefore cost $12c + 8t$. To represent the budget constraints, we can write: $12c + 8t \leq 1,000$.

The solution to this inequality is any pair of c and t that makes the inequality true. In this situation, it is any combination of the pounds of coffee and tea that the café can order without going over the $\$1,000$ budget.

We can try different pairs of c and t to see what combinations satisfy the constraint, but it would be difficult to capture all the possible combinations this way. Instead, we can graph a related equation, $12c + 8t = 1,000$, and then find out which region represents all possible solutions.

Here is the graph of that equation.

To determine the solution region, let's take one point on the line and one point on each side of the line, and see if the pairs of values produce true statements.



A point on the line: $(30, 80)$

$$12(30) + 8(80) \leq 1,000$$
$$360 + 640 \leq 1,000$$
$$1,000 \leq 1,000$$

This is true.

A point below the line: $(20, 40)$

$$12(20) + 8(40) \leq 1,000$$
$$240 + 320 \leq 1,000$$
$$560 \leq 1,000$$

This is true.

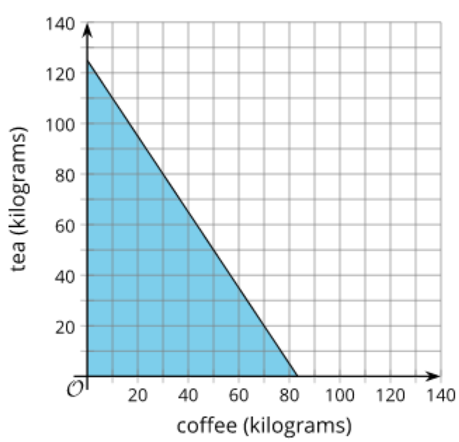
A point above the line: $(70, 100)$

$$12(70) + 8(100) \leq 1,000$$
$$840 + 800 \leq 1,000$$
$$1,640 \leq 1,000$$

This is false.

The points on the line and in the region below the line are solutions to the inequality. Let's shade the solution region.

It is easy to read solutions from the graph. For example, without any computation, we can tell that $(50, 20)$ is a solution because it falls in the shaded region. If the café orders 50 50 kilograms of coffee and 20 20 kilograms of tea, the cost will be less than $\$1,000$.



Cool-down: Weekend of Games (*5 minutes*)

Addressing: NC.M1.A-REI.12

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

Graphing technology should not be used in this cool-down.

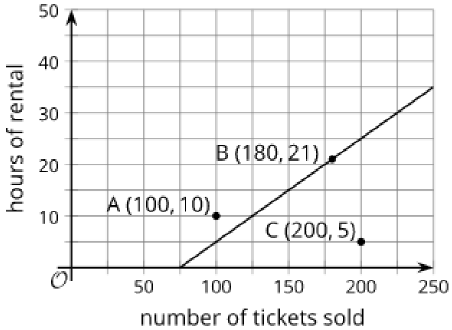
Cool-down

To raise money for after-school programs at an elementary school, a group of parents is holding a weekend of games in a community center. They charge \$8 per person for entry into the event. The group would like to earn at least \$600, after paying for the cost of renting the space, which is \$40 an hour.

1. The line is the graph of $8x–40y = 600$. Select all points (A, B, C) whose (x, y) values represent the group reaching its fundraising goal. Explain or show your reasoning.

2. If x represents the number of entry tickets sold and y the hours of space rental, which inequality represents the constraints in the situation?

   a. $8x–40y < 600$
   b. $8x–40y \leq 600$
   c. $8x–40y > 600$
   d. $8x–40y \geq 600$



Student Reflection:

Imagine you were absent today. You ask a classmate to explain to you why there is shading on the graphs. How might that classmate explain it to you in a way that you would understand? Are there questions you might have for your classmate?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

How did the student work that you selected impact the direction of the discussion? What student work might you pick next time if you taught the lesson again?

Practice Problems

1. To qualify for a loan from a bank, the total in someone's checking and savings accounts together must be $500 or more.

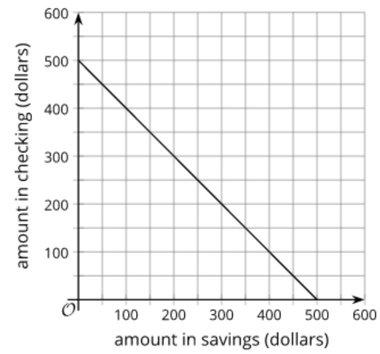
   a. Find three possible combinations of checking and savings account balances that would together amount to $500 or more.

   b. Would someone qualify for a loan if they have $50 in savings and $450 in checking? Why?

   c. Which of these inequalities **best** represents this situation?

      - $x + y < 500$
      - $x + y \leq 500$
      - $x + y > 500$
      - $x + y \geq 500$

   d. To the right is a graph of the line $x + y = 500$. Plot ordered pairs representing the savings and checking account combinations you identified in part a and the combination provided in part b on the graph below. What general region defines where these points are located relative to the graphed line? (example: above, below, on)



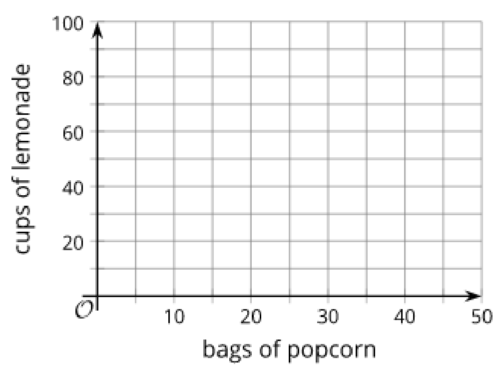
2. The soccer team is selling bags of popcorn for $3 each and cups of lemonade for $2 each. To make a profit, they must collect a total of more than $120.

   a. Write an equation to represent the number of bags of popcorn sold, p, and the number of cups of lemonade sold, c, in order to break even.

   b. Graph the line representing your equation in part a on the coordinate plane.

   c. Explain how we could check if the points above, below, and on the line correspond to the team earning a profit.

   d. Write an inequality to represent the number of bags of popcorn sold, p, and the number of cups of lemonade sold, c, in order to make a profit.



3. Tyler filled a small jar with quarters and dimes and donated it to his school's charity club. The club member receiving the jar asked, "Do you happen to know how much is in the jar?" Tyler said, "I know it's at least $8.50, but I don't know the exact amount."
   a. Write an inequality to represent the relationship between the number of dimes, d, the number of quarters, q, and the dollar amount of the money in the jar.
   b. Identify one solution to this inequality and explain what a solution would mean in this situation.
   c. Suppose Tyler knew there are 25 dimes in the jar. Write an inequality that represents how many quarters could be in the jar.
4. Square $ABCD$ is drawn on the coordinate plane with vertex A at $(-4,-2)$ and the midpoint of side AB at $(-1,-6)$. What is the area of the square?

   (From Unit 3, Lessons 13 & 14)
5. Kiran says, "I bought 2.5 pounds of red and yellow lentils. Both were $1.80 per pound. I spent a total of $4.05."
   a. Write a system of equations to describe the relationships between the quantities in Kiran's statement. Be sure to specify what each variable represents.
   b. Elena says, "That can't be right." Explain how Elena can tell that something is wrong with Kiran's statement.
   c. Kiran says, "Oops, I meant to say I bought 2.25 pounds of lentils." Revise your system of equations to reflect this correction.
   d. Is it possible to tell for sure how many pounds of each kind of lentil Kiran might have bought? Explain your reasoning.

   (From Unit 3, Lesson 12)
6. Andre is solving the inequality $14x+3\leq 8x+3$. He first solves a related equation.

   $$14x+3=8x+3$$
   $$14x=8x$$
   $$8=14$$

   This seems strange to Andre. He thinks he probably made a mistake. What was his mistake?

   (From Unit 2)
7. Here is an inequality: $-7-(3x+2)<-8(x+1)$

   Select **all** the values of x that are solutions to the inequality.
   a. $x=-0.2$
   b. $x=-0.1$
   c. $x=0$
   d. $x=0.1$
   e. $x=0.2$
   f. $x=0.3$

   (From Unit 2)
8. Solve this inequality: $\frac{x-4}{3}\geq\frac{x+3}{2}$

   (From Unit 2)
9. At a store, a shirt was marked down in price by $10.00. A pair of pants doubled in price. Following these changes, the price of every item in the store was cut in half. Write two different expressions that represent the new cost of the items, using s for the cost of each shirt and p for the cost of a pair of pants. Explain the different information each one shows.[2]

   (Addressing NC.6.EE.2)

[2] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds.

Lesson 16: Graphing Linear Inequalities in Two Variables (Part Two)

PREPARATION

Lesson Goals	Learning Targets
• Given the graph of a related equation, determine the solution region to an inequality in two variables by testing the points on the line and on either side of the line. • Understand that the solutions to a linear inequality in two variables are represented graphically as a half-plane bounded by a line.	• Given a two-variable inequality and the graph of the related equation, I can determine which side of the line the solutions to the inequality will fall. • I can describe the graph that represents the solutions to a linear inequality in two variables.

Lesson Narrative

In the previous lesson, students examined two-variable inequalities in context. They saw that there are multiple solutions to a two variable inequality and that those solutions fall on one side of the graph of a boundary line. Students tested points, used the structure of the inequalities, and reasoned about the context to identify the correct region. In this lesson, students graph inequalities without a context, without the real-world situation to guide their sense of what is plausible. Students retain the non-contextual strategies from the previous lesson, and attend to which strategies will be most useful in a given problem. In doing so, they reason abstractly and quantitatively. (MP2)

Working with pure equations allows students to encounter strict inequalities. In that case, students learn, points on the boundary line are not included in the solution region. We indicate that by making the boundary line dashed rather than solid.

What teaching strategies will you be focusing on during this lesson?

Focus and Coherence

Addressing
NC.M1.A-REI.12: Represent the solutions of a linear inequality or a system of linear inequalities graphically as a region of the plane.

Agenda, Materials, and Preparation

- **Warm-up** (*10 minutes*)
  - *Optional:* Sticky notes and/or mini whiteboards for student supports
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U3.L16 Cool-down (print 1 copy per student)

LESSON

Warm-up: Less Than, Equal to, or More Than 6? (*10 minutes*)

Instructional Routines: Math Talk; Discussion Supports (MLR8) - Responsive Strategy
Building Towards: NC.M1.A-REI.12

In the first activity of the lesson, students engage in a *Math Talk* to consider whether the expression $-2x+3y$ is greater than, less than, or equal to 6 for given (x,y) pairs. This warm-up strengthens the computation and reasoning that students need to determine the solution region of a linear inequality in two variables.

Students could reason about the answers by considering the signs and relative sizes of the x- and y-values, rather than performing full computations. This is an opportunity to notice and make use of structure (MP7).

RESPONSIVE STRATEGY

To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Step 1

- Display one problem at a time. Give students quiet time for each problem and ask them to give a signal when they have an answer and a strategy.
- Keep all problems displayed throughout the talk.

Student Task Statement

Here is an expression: $-2x+3y$. Decide if the values in each ordered pair, (x,y), make the value of the expression less than, greater than, or equal to 6.

1. $(2,4)$
2. $(0,2)$
3. $(-1,1)$
4. $(-2,1)$

Step 2

- Facilitate a whole-class discussion by asking students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:
  - “Who can restate ______’s reasoning in a different way?”
  - “Did anyone have the same strategy but would explain it differently?”
  - “Did anyone solve the problem in a different way?”
  - “Does anyone want to add on to ______’s strategy?”
  - “Do you agree or disagree? Why?”

RESPONSIVE STRATEGY

Display sentence frames to support students when they explain their strategy. For example, “First, I ____ because....” or “I noticed ____ so I....” Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Discussion Supports (MLR8)

- To help students recall the meaning of a solution to an inequality, ask: "Which pairs, if any, are solutions to the inequality $-2x+3y \le 6$?" Make sure students recognize that both $(0,2)$ and $(-1,1)$ are solutions because they make the inequality true.

DO THE MATH	PLANNING NOTES

Activity 1: Finding the Solution Region (*15 minutes*)

Instructional Routines: Poll the Class; Discussion Supports (MLR8) - Responsive Strategy
Building Towards: NC.M1.A-REI.12

In this activity, students begin by working collaboratively to find the region in the plane that contains the solutions to an inequality. Since each pair of students will test their own points, the collective graph of the class’s solutions will reinforce the idea that all of the solutions to an inequality lie on one side of a boundary line.

Students then discuss strategies for choosing points to test or otherwise determining which side of the boundary line contains the solutions to an inequality. Finally, students will practice graphing inequalities on their own.

This is the first time in the course that students will participate in a *Poll the Class* routine.

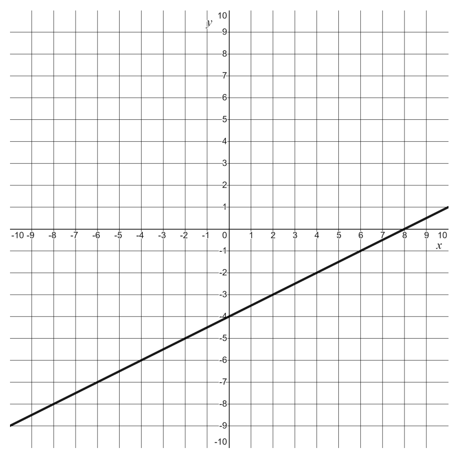
POLL THE CLASS	***What Is This Routine?*** This routine is used to register an initial response or an estimate, most often at the beginning of an activity or discussion. It can also be used when it is important to collect data from each student in class; for example, "What is the length of your ear in centimeters?" Every student in class reports a response to the prompt. Teachers need to develop a mechanism by which poll results are collected and displayed so that this frequent form of classroom interaction is seamless. Smaller classes might be able to conduct a roll call by voice. For larger classes, students might be given mini-whiteboards or a set of colored index cards to hold up. Free and paid commercial tools are also readily available.

	Why This Routine? Collecting data from the class to use in an activity with the *Poll the Class* routine makes the outcome of the activity more interesting. In other cases, going on record with an estimate makes people want to know if they were right and increases investment in the outcome. If coming up with an estimate is too daunting, ask students for a guess that they are sure is too low or too high. Putting some boundaries on possible outcomes of a problem is an important skill for mathematical modeling (MP4).

Step 1

- Display the inequality $x - 2y \leq 8$ for all to see, along with a coordinate grid showing a graph of the line $x - 2y = 8$.
- Ask students what must be true about the points that are on the line. (They are solutions to the equation $x - 2y = 8$; they are points with coordinates where $x - 2y = 8$.) If needed, test a point on the line such as $(10, 1)$ to confirm this.
- Have students work in partners to test at least one point above the line and at least one point below the line, then graph the inequality.

Step 2



- After a few minutes, use *Poll the Class* to collect all points students found that satisfy the inequality.
- Plot several of these points.
- Ask students how they decided which points to test.
  - “Which points are easiest to test?” (The point $(0, 0)$ if it is not on the line, or points where the x- or y-coordinate is zero)
  - “Are there any points that are not useful to test?” (points on the line)

Step 3

- Ask students to arrange themselves in small groups or use visibly random groupings.
- Provide students with 3-4 minutes of quiet work time to graph the inequalities and then additional time to discuss their strategies and solutions with their group.
- Depending on time available, assign two or three inequalities to each group.

Monitoring Tip: Monitor for students who test “strategic” points like $(0, 0)$, and for students who reason about the structure of the inequality (“if $-2y \geq -4$, we have to use negative values for y to make a larger number”).

Let these students know that they may be asked to share later. Include at least one student who does not typically volunteer.

***Advancing Student Thinking*:** Some students will use reasoning like, “the inequality says the expression is less than so I shaded below the line." Refer these students to the inequality graphed on the display, as well as the inequality $x \geq y$.

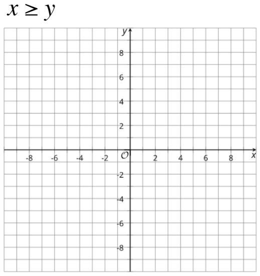
The inequality $3x \leq 0$ may be difficult for students who do not remember graphs of vertical lines. If students struggle with the form of the related equation or graph the line $y = 3x$ instead, ask them if the point $(1, 3)$ satisfies the equation $3x = 0$. Invite them to come up with some points that do satisfy this equation. Reassure them that it’s okay for these points to have y-coordinates even though there is no y-variable in the equation. Only the x-coordinate will be substituted into the expression $3x$.

Student Task Statement

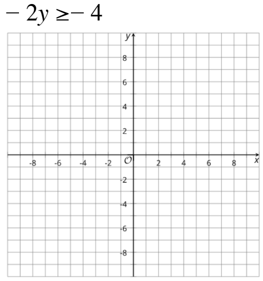
Here are four inequalities. Study each inequality assigned to your group and work with your group to:

- Graph the related equation
- Determine the side of the line that has the solutions to the inequality
- Shade the solution region

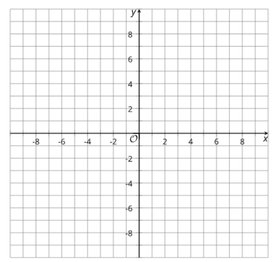
$x \geq y$



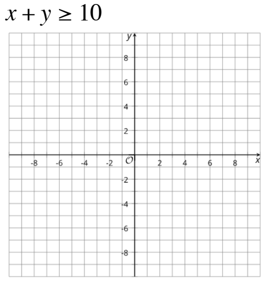
$-2y \geq -4$



$3x \leq 0$



$x + y \geq 10$



Step 4

- Display four graphs that are representative of students' work.
- Ask previously identified students to share their methods for determining the solution region.

In the next activity, students will take a closer look at whether the boundary line itself is part of the solution region. For now, it is sufficient that students see that the graph of an equation that is related to each inequality delineates the solution and non-solution regions.

RESPONSIVE STRATEGY

Use this routine to support whole-class discussion. For each observation that is shared, ask students to restate what they heard using precise mathematical language. Consider providing students time to restate what they hear to a partner before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This provides more students with an opportunity to produce language as they interpret the reasoning of others.

Discussion Supports (MLR8)

DO THE MATH

PLANNING NOTES

Activity 2: Sketching Solutions to Inequalities (*10 minutes*)

Instructional Routine: Stronger and Clearer Each Time (MLR1)	
Addressing: NC.M1.A-REI.12	

So far, students have looked at the regions that represent solutions to inequalities. They recognized that the boundary between the two regions is the graph of an equation that is related to the inequality. Students did not, however, look closely at whether the boundary line itself is a part of the solution. That investigation is the focus of this activity.

Students reason with algebraic and graphical representations of inequalities in two directions. They first graph the solutions to given inequalities, and later write inequalities whose solutions could be represented by given graphs.

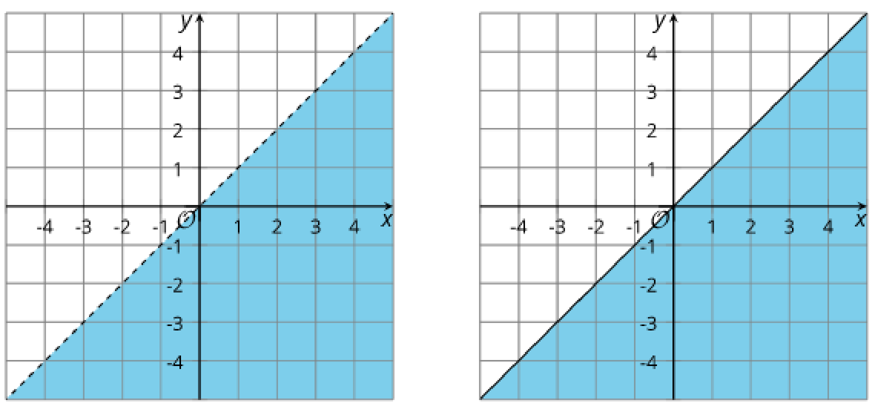
If many students get stuck on graphing or writing inequalities, consider moving fairly quickly through the activity and using the discussion questions in the Lesson Debrief to help students gain clarity and focus.

Step 1

- Display the inequalities $x \geq y$ and $x > y$ for all to see. Then, ask students to consider whether the following coordinate pairs are solutions to each inequality.

    1. $(5,4)$
    2. $(5,4.9)$
    3. $(5,5)$

- Make sure students understand why all three coordinate pairs are solutions to $x \geq y$, but only $(5,4)$ and $(5,4.9)$ are solutions to $x > y$. Display two graphs, each representing one of these inequalities.



- Ask students to predict which graph represents which inequality. Consider quickly polling the class on their predictions.

- Explain that the solid line is a way to say that all the points on that line ($x = y$) are solutions, and the dashed line is a way to say otherwise. (This is similar to how we use solid and open circles to represent the boundary values of a one-variable inequality on a number line.)

- Because the solutions to $x > y$ do not include coordinates where x and y are equal, the graph of $x = y$ is drawn with a dashed line. The solutions to $x \geq y$ do include coordinates where x and y are equal, so the graph of $x = y$ is drawn with a solid line.

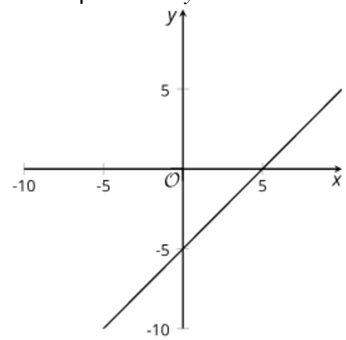
Step 2

Tell students they will now sketch the solutions of some other inequalities and think about whether or not the boundary line is included in the solutions.

Monitoring Tip: Select students to share their sketched graphs for the first set of questions and the inequalities they wrote for the second question. Use their work and explanations to help the class synthesize the new ideas in this lesson.

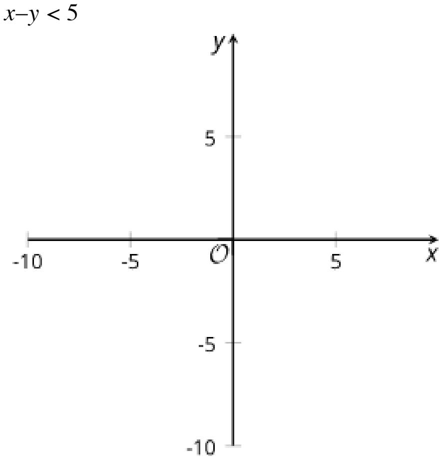
Student Task Statement

1. Here is a graph that represents solutions to the equation $x - y = 5$.

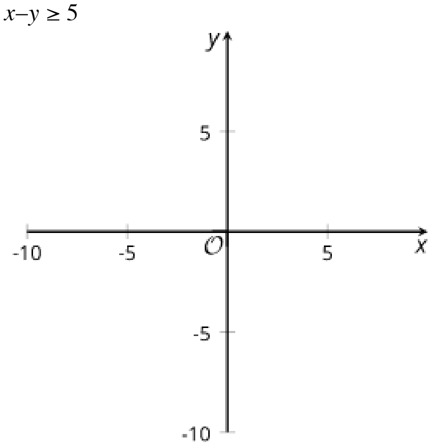


Sketch two quick graphs representing the solutions to each of these inequalities:

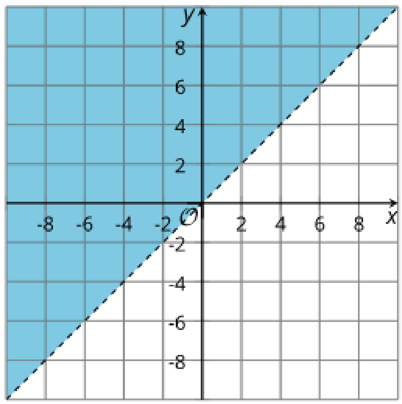
$x - y < 5$



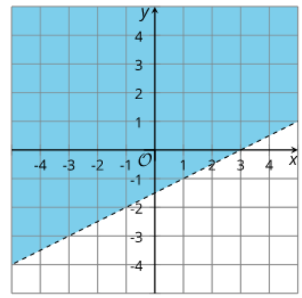
$x - y \geq 5$



2. For the graph below, write an inequality whose solutions are represented by the shaded part of the graph.



Are You Ready For More?



1. The points $(7,3)$ and $(7,5)$ are both in the solution region of the inequality $x-2y<3$.
   a. Compute $x-2y$ for both of these points.
   b. Which point comes closest to satisfying the equation $x-2y=3$? That is, for which (x,y) pair is $x-2y$ closest to 3?
2. The points $(3,2)$ and $(5,2)$ are also in the solution region. Which of these points comes closest to satisfying the equation $x-2y=3$?
3. Find a point in the solution region that comes even closer to satisfying the equation $x-2y=3$. What is the value of $x-2y$?
4. For the points $(5,2)$ and $(7,3)$, $x-2y=1$. Find another point in the solution region for which $x-2y=1$.
5. Find $x-2y$ for the point $(5,3)$. Then find two other points that give the same answer.

Step 3

- Use the *Stronger and Clearer Each Time* routine to give students a structured opportunity to refine their reasoning and language about the meaning of boundary lines that are and are not part of the solution to an inequality.
  - Give students 1 minute to jot down some ideas in response to the question, “How did you determine where the solution regions are for the two inequalities in question 1?”
  - Have students pair up for 1 minute, taking turns reading their ideas, and giving each other feedback and additional ideas (1 minute total).
  - Give students 1 minute to revise their writing, using feedback and additional ideas from their partner, to create a second draft that is stronger and clearer than their initial response.

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is for students to gain comfort in graphing two-variable inequalities without an associated context. Students pay special attention to the ordered pairs on the boundary line, which is the focus of this debrief.

Refer to the work students have done in the last activity. Discuss with students how they made decisions about the solution region and boundary line for the given inequalities, and about the inequality symbol for the given graphs. Ask questions such as:

- "Once you knew where the boundary line is, how did you decide which side of the line represents the solution region?"
- "How did you decide whether the boundary line should be solid or dashed?"
- "When you saw the graph showing the solution region, how did you determine the inequality symbol to use?"

Some students might incorrectly conclude that an inequality with a < symbol will be shaded below the boundary line and that an inequality with a > symbol will be shaded above it. The inequalities in the first question of Activity 2 can be used to show that this is not the case.

Take $x - y < 5$, for example. We're looking for coordinate pairs that produce a value of less than 5 when y is subtracted from x. Let's see if $(0,0)$ meets this condition: $0 - 0 < 5$ gives $0 < 5$, which is a true statement. This means that $(0,0)$, which is above the graph of $x - y = 5$, is in the solution region. If we test a point below the line: say, $(10, -10)$, we would see that $x - y$ is greater than 5, not less than 5. This means that the region below the line is for non-solutions.

Emphasize that we cannot assume that the < or $\leq$ symbol means shading below a line. It is important to test points on either side of the line to see if the pair of values make the inequality true, or to reason carefully about the inequality statement and think about pairs of values that would satisfy the inequality.

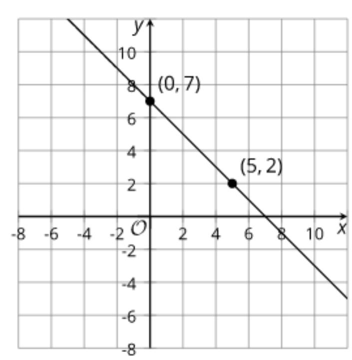
PLANNING NOTES

Student Lesson Summary and Glossary

The equation $x + y = 7$ is an equation in two variables. Its solution is any pair of x and y whose sum is 7. The pairs $x = 0,\ y = 7$ and $x = 5,\ y = 2$ are two examples.

We can represent all the solutions to $x + y = 7$ by graphing the equation on a coordinate plane.

The graph is a line. All the points on the line are solutions to $x + y = 7$.

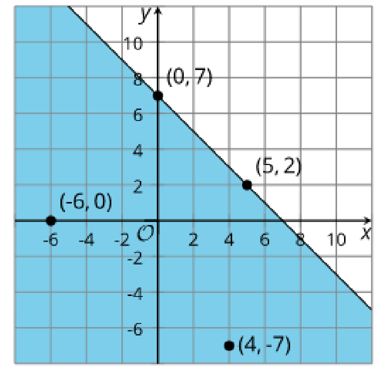


The inequality $x + y \leq 7$ is an inequality in two variables. Its solution is any pair of x and y whose sum is 7 or less than 7.

This means it includes all the pairs that are solutions to the equation $x + y = 7$, but also many other pairs of x and y that add up to a value less than 7. The pairs $x = 4,\ y = -7$ and $x = -6,\ y = 0$ are two examples.

On a coordinate plane, the solution to $x + y \leq 7$ includes the line that represents $x + y = 7$. If we plot a few other (x, y) pairs that make the inequality true, such as $(4, -7)$ and $(-6, 0)$, we see that these points fall on one side of the line. (In contrast, (x, y) pairs that make the inequality false fall on the other side of the line.)

We can shade that region on one side of the line to indicate that all points in it are solutions.



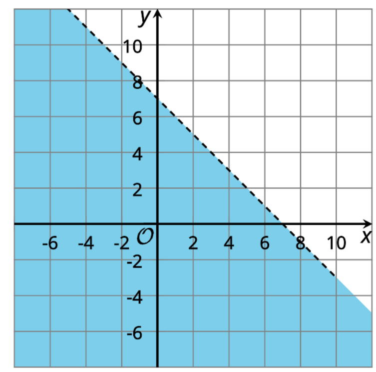
What about the inequality $x + y < 7$?

The solution is any pair of x and y whose sum is less than 7. This means pairs like $x = 0,\ y = 7$ and $x = 5,\ y = 2$ are *not* solutions.

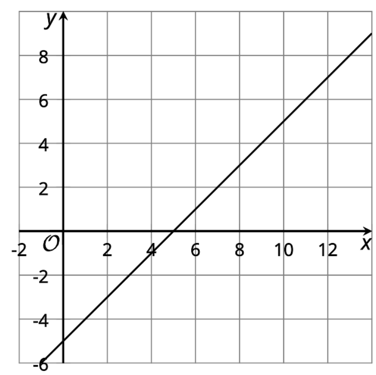
On a coordinate plane, the solution does not include points on the line that represent $x + y = 7$ (because those points are x and y pairs whose sum is 7).

To exclude points on that boundary line, we can use a dashed line.

All points below that line are (x, y) pairs that make $x + y < 7$ true. The region on that side of the line can be shaded to show that it contains the solutions.



When identifying the solution region, it is important *not* to assume that the solution will be above the line because of a ">" symbol or below the line because of a "<" symbol. For example, when graphing the inequality $x - y \geq 5$ we would start by graphing the related equation $x - y = 5$:



Points above the line such as $(0, 0)$ are *not* solutions to the inequality because because the (x, y) pairs make the inequality false. Points that are on or below the lines are solutions, so we can shade that lower region.

Cool-down: Pick a Graph (*5 minutes*)

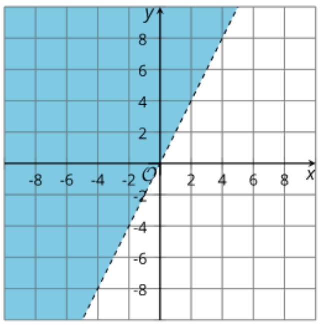
Addressing: NC.M1.A-REI.12

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize in the next lesson to support students in advancing their current understanding.

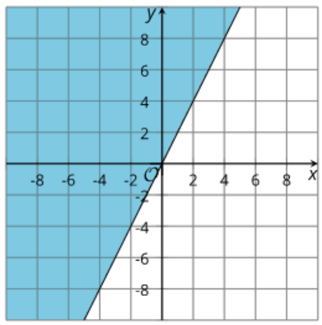
Cool-down

1. The line in each graph represents $y = 2x$. Which graph represents $2x > y$?

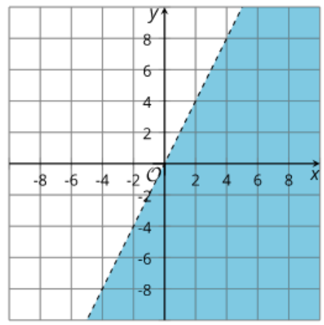
Graph A



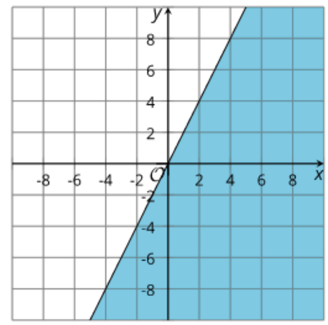
Graph B



Graph C



Graph D



2. Explain your reasons for choosing that graph.

Student Reflection: After having experienced linear equations and inequalities, which do you feel more confident with and why?

What makes you less confident with the other?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

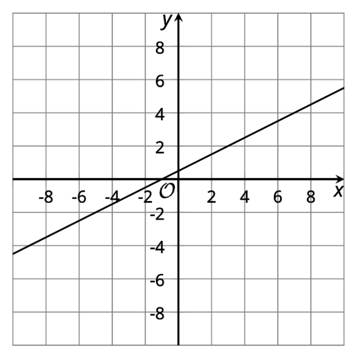
TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Which students had opportunities to share their diagrams and thinking during whole-class discussion? How did you select these students?

Practice Problems

1. Here is a graph of the equation $2y - x = 1$.



   a. Are the points $(0, \frac{1}{2})$ and $(-7, -3)$ solutions to the equation? Explain or show how you know.

   b. Check if each of these points is a solution to the inequality $2y - x > 1$:

   $(0, 2)$ $\quad$ $(8, \frac{1}{2})$ $\quad$ $(-6, 3)$ $\quad$ $(-7, -3)$

   c. Shade the region that represents the solution set to the inequality $2y - x > 1$.

   d. Are the points on the line included in the solution set? Explain how you know.

2. Select **all** coordinate pairs that are solutions to the inequality $5x + 9y < 45$.

   a. $(0, 0)$
   b. $(5, 0)$
   c. $(9, 0)$
   d. $(0, 5)$
   e. $(0, 9)$
   f. $(5, 9)$
   g. $(-5, -9)$

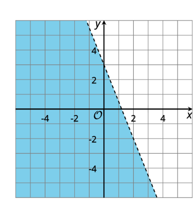
3. Consider the linear equation $2y - 3x = 5$.

   a. The pair $(-1, 1)$ is a solution to the equation. Find another (x, y) pair that is a solution to the equation.

   b. Are $(-1, 1)$ and $(4, 1)$ solutions to the inequality $2y - 3x < 5$? Explain how you know.

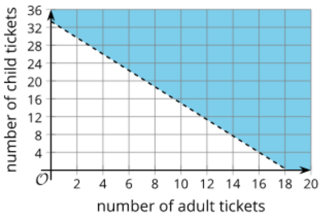
   c. Explain how to use the answers to the previous questions to graph the solution set to the inequality $2y - 3x < 5$.

4. The boundary line on the graph represents the equation $5x + 2y = 6$. Write an inequality that is represented by the graph.

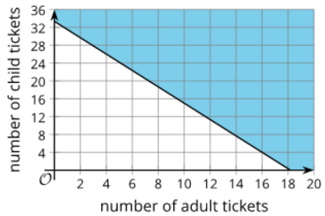


5. Tickets to the aquarium are \$11 for adults and \$6 for children. An after-school program has a budget of \$200 for a trip to the aquarium. If the boundary line in each graph represents the equation $11x + 6y = 200$, which graph represents the cost constraint in this situation?

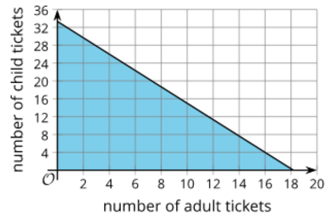
Graph A



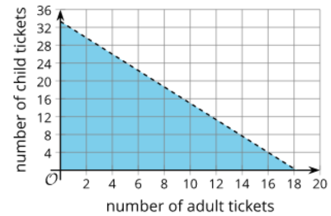
Graph B



Graph C



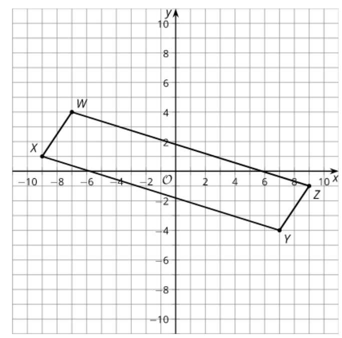
Graph D



6. Diego claims that he can tell, using slopes, that this quadrilateral is a parallelogram. Noah looks at the slopes Diego calculated and says he could be even more specific and call it a rectangle.

   Do you agree with either of them? Explain or show your reasoning.

   (From Unit 3, Lesson 8)



7. Solve each system of equations without graphing.

   a. $\begin{cases} 4d + 7e = 68 \\ -4d - 6e = -72 \end{cases}$

   b. $\begin{cases} \frac{1}{4}x + y = 1 \\ \frac{3}{2}x - y = \frac{4}{3} \end{cases}$

   (From Unit 3, Lesson 10)

8. Mai and Tyler are selling items to earn money for their elementary school. The school earns w dollars for every wreath sold and p dollars for every potted plant sold. Mai sells 14 wreaths and 3 potted plants and the school earns \$70.50. Tyler sells 10 wreaths and 7 potted plants and the school earns \$62.50.

   This situation is represented by this system of equations: $\begin{cases} 14w + 3p = 70.50 \\ 10w + 7p = 62.50 \end{cases}$

   Explain why it makes sense in this situation that the solution of this system is also a solution to $4w + (-4p) = 8.00$.

   (From Unit 3, Lesson 11)

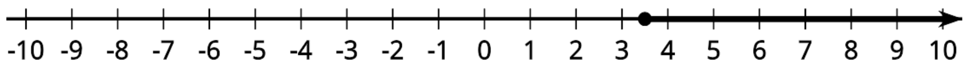
9. Elena is planning to go camping for the weekend and has already spent \$40 on supplies. She goes to the store and buys more supplies.

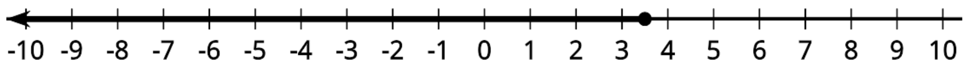
   Which inequality represents d, the total amount in dollars that Elena spends on supplies?

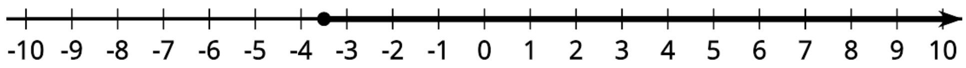
   a. $d > 40$
   b. $d \geq 40$
   c. $d < 40$
   d. $d \leq 40$

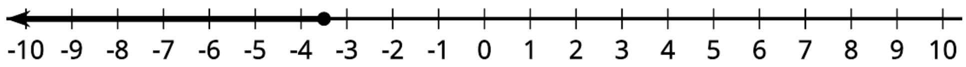
   (From Unit 2)

10. Which graph represents the solution to $\frac{4x-8}{3} \leq 2x - 5$?

a. 

b. 

c. 

d. 

(From Unit 2)

11. Solve $-x < 3$. Explain how to find the solution set.

(From Unit 2)

Lesson 17: Solving Problems with Inequalities in Two Variables

PREPARATION

Lesson Goals	Learning Targets
• Identify an inequality, a graph, an ordered pair, and a description that represent the constraints and possible solutions in a situation. • Understand that a constraint on two variables can be represented by an inequality, a graph (a half-plane), and a verbal description. • Write inequalities in two variables to represent the constraints in a situation and use technology to graph the solution set to answer questions about the situation.	• I can use graphing technology to find the solution to a two-variable inequality. • When given inequalities, graphs, and descriptions that represent the constraints in a situation, I can connect the different representations and interpret them in terms of the situation.

Lesson Narrative

By now students recognize that solutions to linear inequalities can be found by graphing, and that this can be done by first graphing a related equation and deciding on the solution region. In this lesson, they learn to use graphing technology to find the solution set of a linear inequality in two variables.

Students then use this skill to solve problems that involve inequalities. They write linear inequalities to represent the constraints in situations and then use the representations (including the graphs of the solutions) to answer questions about the situations. As they write inequalities from descriptions, decide on the solution sets, and interpret points in a solution region, students engage in quantitative and abstract reasoning (MP2).

What are you excited for your students to be able to do after this lesson?

Focus and Coherence

Building On	Addressing
NC.7.EE.4: Use variables to represent quantities to solve real-world or mathematical problems. **b**. Construct inequalities to solve problems by reasoning about the quantities. ● Fluently solve multi-step inequalities with the variable on one side, including those generated by word problems. ● Compare an algebraic solution process for equations and an algebraic solution process for inequalities. ● Graph the solution set of the inequality and interpret in context.	**NC.M1.A-CED.1:** Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems. **NC.M1.A-REI.12:** Represent the solutions of a linear inequality or a system of linear inequalities graphically as a region of the plane.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*10 minutes*)
  - Graphing technology is required. Acquire devices that can access Desmos (recommended) or other graphing technology. It is ideal if each student has their own device.
- **Activity 1** (*20 minutes*)
  - Desmos access
- **Activity 2** (*Optional, 15 minutes*)
  - Representations of Inequalities card sort (print 1 copy per every 2 students and cut up in advance)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U3.L17 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.7.EE.4b

The purpose of this bridge is to focus on application of mathematics, specifically inequalities, in a real-world context.

Student Task Statement[1]

Elena is participating in a fundraiser at school. She will receive donations from two people. A cousin will donate \$0.40 for every $\frac{1}{8}$ mile that Yuri walks. A friend will give Elena a one-time donation of \$30.

What is the minimum number of miles Elena needs to walk to raise at least \$50?

DO THE MATH	PLANNING NOTES

[1] Adapted from Achievethecore.org

Warm-up: Graphing Inequalities with Technology (*10 minutes*)

Instructional Routine: Graph It	
Addressing: NC.M1.A-REI.12	

In this *Graph It* warm-up, students use Desmos to graph simple linear inequalities in two variables. They practice adjusting the graphing window until the solution regions become visible and give useful information. Later in the lesson, students will write inequalities that represent constraints in different situations and find the solution sets. The exercises here prepare students to do the latter using Desmos.

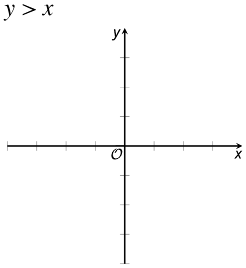
Step 1

- Give students access to graphing technology. If using Desmos:
  - Explain to students that typing "<= " gives the ≤ symbol and typing ">=" gives the ≥ symbol.
  - Remind students that the + and – buttons can be used to zoom in and out of the graphing window, and that the wrench button in the upper right corner can be used to set the graphing window precisely.
- If using other graphing technology available in your classroom:
  - Demonstrate how to enter the ≤ and ≥ symbols.
  - Remind students how to set a useful graphing window by zooming in or out, and how to set a precise graphing window by specifying the horizontal and vertical boundaries.
  - (For technology that takes only equations or inequalities in slope-intercept form:) Remind students that some inequalities might need to be rewritten such that y is isolated before the inequality can be entered into the graphing tool.

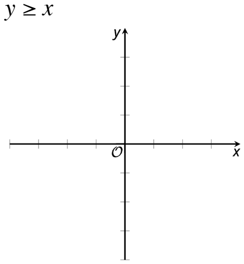
Student Task Statement

Access www.desmos.com/calculator to graph the solution region of each inequality and sketch each graph. Adjust the graphing window as needed to show meaningful information.

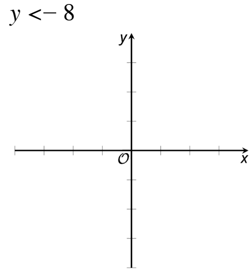
1. $y > x$



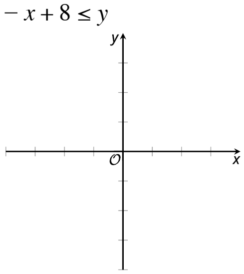
2. $y \geq x$



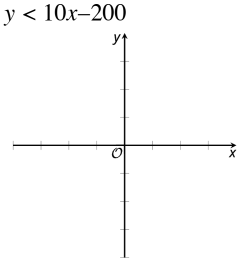
3. $y < -8$



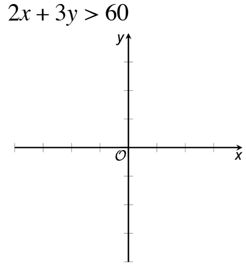
4. $-x + 8 \leq y$



5. $y < 10x - 200$



6. $2x + 3y > 60$



Step 2

- Display the correct solution regions for all to see and ask students to check their graphs. Discuss any challenges students may have come across when trying to graph using technology.
- Explain to students that they will now use Desmos to find solutions to some inequalities that represent constraints in situations.

DO THE MATH	PLANNING NOTES

Activity 1: Solving Problems with Inequalities in Two Variables (*20 minutes*)

Instructional Routines: Graph It; Aspects of Mathematical Modeling; Compare and Connect (MLR7)
Addressing: NC.M1.A-CED.1; NC.M1.A-REI.12

This *Graph It* activity enables students to integrate several ideas and skills from the past few lessons. Students write inequalities in two variables to represent constraints in situations, use technology to graph the solutions, interpret points in the solution regions, and use the inequalities and the graphs to answer contextual questions. In doing so, they engage in *Aspects of Mathematical Modeling* (MP4).

The questions in this activity are written in pairs. The same constraints and contexts will be used in an upcoming lesson on systems of linear inequalities in two variables.

Decide on the structure for the activity depending on the time available and the amount of practice each student needs. Here are some possibilities:

- Assigning all three pairs of questions to all students.
- Assigning each student a pair of questions, arranging students who work on different pairs in groups of three, and asking them to explain their solutions to one another.
- Arranging students in groups of three, assigning the same pair of questions to each group, and—for each pair of questions—asking one group to present the solutions to the class.

Step 1

- Continue to provide access to Desmos. Explain that students will now write and graph inequalities to solve problems about some situations.
- Assign at least one pair of questions about the same context to each student. See above for some possible ways to structure the activity.
- Some students might not be familiar with terms such as "savings," "checking," or "premium." Explain any unfamiliar terms as needed.

RESPONSIVE STRATEGY

Leverage choice around perceived challenge. Chunking this task into more manageable parts may also support students who benefit from additional processing time. Invite students to select one of the situations to work on. If students are arranged in groups, each member should be prepared to share during the whole-class discussion.

Supports accessibility for: Attention; Social-emotional skills

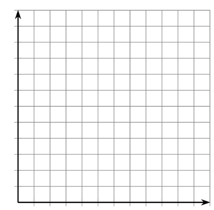
***Advancing Student Thinking*:** One inequality in the bank account context involves only one variable. Some students might think that all inequalities they write must include two variables. Reassure them that this might not always be the case. Consider pointing to examples from earlier activities in which they wrote or graphed inequalities such as $y > 2$ or $b < 10$.

Student Task Statement

Here are three situations (bank accounts, concert tickets, and advertising packages). There are two questions about each situation. For each question that you work on:

a. Write an inequality to describe the constraints. Specify what each variable represents.
b. Use Desmos to graph the inequality. Sketch the solution region on the coordinate plane and label the axes.
c. Name one solution to the inequality and explain what it represents in that situation.
d. Answer the question about the situation.

Bank Accounts



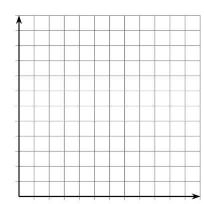
1. A customer opens a checking account and a savings account at a bank. They will deposit a maximum of $600, some in the checking account and some in the savings account. (They might not deposit all of it and instead keep some of the money as cash.)

   If the customer deposits $200 in their checking account, what can you say about the amount they deposit in their savings account?

2. The bank requires a minimum balance of $50 in the savings account. It does not matter how much money is kept in the checking account.

   If the customer deposits no money in the checking account but is able to maintain both accounts without penalty, what can you say about the amount deposited in the savings account?

Concert Tickets



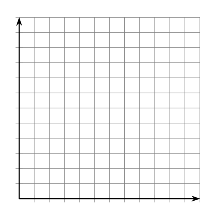
1. Two kinds of tickets to an outdoor concert were sold: lawn tickets and seat tickets. Fewer than 400 tickets in total were sold.

   If you know that exactly 100 lawn tickets were sold, what can you say about the number of seat tickets?

2. Lawn tickets cost $30 each and seat tickets cost $50 each. The organizers want to make at least $14,000 from ticket sales.

   If you know that exactly 200 seat tickets were sold, what can you say about the number of lawn tickets?

Advertising Packages



1. An advertising agency offers two packages for small businesses who need advertising services. A basic package includes only design services. A premium package includes design and promotion. The agency's goal is to sell at least 60 packages in total.

   If the agency sells exactly 45 basic packages, what can you say about the number of premium packages it needs to sell to meet its goal?

2. The basic advertising package has a value of $1,000 and the premium package has a value of $2,500. The goal of the agency is to sell more than $60,000 worth of small-business advertising packages.

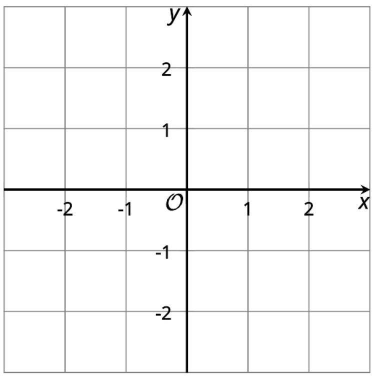
   If you know that exactly 10 premium packages were sold, what can you say about the number of basic packages the agency needs to sell to meet its goal?

Are You Ready For More?

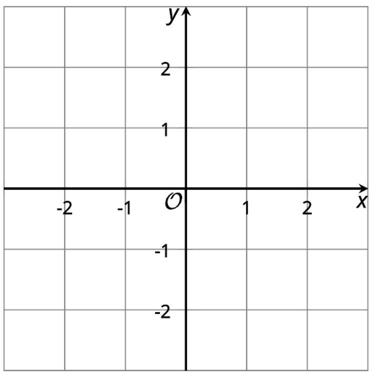
This activity will require a partner.

1. Without letting your partner see it, write an equation of a line so that both the x-intercept and the y-intercept are each between -3 and 3. Graph your equation on one of the coordinate systems.

Your inequality



Your partner's inequality



2. Still without letting your partner see it, write an inequality for which your equation is the related equation. In other words, your line should be the boundary between solutions and non-solutions. Shade the solutions on your graph.

3. Take turns stating coordinates of points. Your partner will tell you whether your guess is a solution to their inequality. After each partner has stated a point, each may guess what the other's inequality is. If neither guesses correctly, play continues. Use the other coordinate system to keep track of your guesses.

Step 2

Select one student or one group to present their solutions for each pair of questions and use the *Compare and Connect* routine to focus the discussion on differences and similarities in how students used the inequalities and graphs to help answer the question about each situation.

- Students who worked on the same questions might end up with different graphs and answers because they wrote different inequalities (which might not correctly represent the constraints in the situation). If this happens, ask students to carefully analyze the different inequalities and look for the potential causes for the discrepancy.
- If there are different graphs and answers, students may determine that the wrong symbols or the wrong numbers were entered into the graphing tool. Another possibility might be that students made different decisions about the quantities being assigned to the vertical and horizontal axes. In that case, both versions of the graphs might be correct, but the answers to questions might be different if one of the graphs is not interpreted correctly.

DO THE MATH	PLANNING NOTES

Activity 2: Representations of Inequalities (*Optional, 15 minutes*)

Instructional Routines: Card Sort; Take Turns
Addressing: NC.M1.A-CED.1; NC.M1.A-REI.12

This optional *Card Sort* activity allows students to practice interpreting inequalities in context and reasoning about their solutions graphically and numerically. A sorting and matching task gives students opportunities to analyze representations, statements, and structures closely and to make connections (MP2, MP7). Review the Representations of Inequalities card sort for planning purposes.

As students use the *Take Turns* routine to explain their thinking to a partner, encourage them to use precise language and mathematical terms to refine their explanations (MP6).

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping. Distribute one set of pre-cut slips or cards to each pair.
- Ask students to take turns finding a group of four cards that represent the same situation and explaining how they know the representations belong together. Emphasize that while one partner explains, the other should listen carefully, and they should discuss any disagreements.

RESPONSIVE STRATEGY

Provide a checklist that focuses on increasing the length of on-task orientation in the face of distractions. For example, use the directions to create a bulleted checklist of the steps. Invite students to check off the steps at the completion of each phase of the activity. An example list may include items such as: taking turns with matching, two explanations for each partner, two responses/rebuttals for each partner, and four agreed upon final responses as a team.

Supports accessibility for: Attention; Social-emotional skills

Student Task Statement

Your teacher will give you a set of cards. Take turns with your partner to match a group of four cards that contain: a situation, an inequality that represents it, a graph that represents the solution region, and a solution written as a coordinate pair.

For each match that you find, explain to your partner how you know it's a match. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

Record your matches.

Group 1	Group 2	Group 3	Group 4
• situation: perimeter of a rectangle • inequality: • a solution: • sketch of graph:	• situation: jar of coins • inequality: • a solution: • sketch of graph:	• situation: honey and jam • inequality: • a solution: • sketch of graph:	• situation: a school trip • inequality: • a solution: • sketch of graph:

Step 2

- Ask groups of students to volunteer to share their results and explain their rationales. After a group explains why they believe a set of cards belongs together, ask if other groups reasoned about the matches the same way or if they approached the matching differently.
- Attend to the language that students use in their explanations by giving them opportunities to describe the inequalities, graphs, or solutions more precisely.

DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is for students to become more comfortable writing inequalities to represent real-world situations and using their graphs to solve problems. Students learned to use graphing technology to produce the graphs so that the focus could be on these more conceptual skills.

Summarize the lesson by discussing students' work for the last activity (about bank accounts, concert tickets, and advertising packages) and inviting them to reflect on their reasoning process. Discuss questions such as:

- "Of the four things you were asked to do in the last activity—writing an inequality, graphing the solutions, identifying and interpreting a particular solution, and answering the question about the situation—which one did you find most challenging or prone to error?"
- "How is graphing linear inequalities using technology similar to graphing them by hand? How is it different?"
- "You have previously used technology to graph linear equations in two variables. How is using technology to graph inequalities different than using technology to graph equations?"

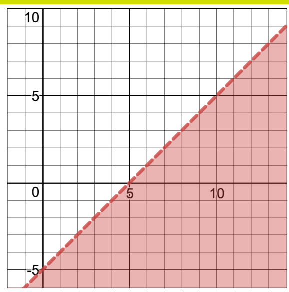
PLANNING NOTES

Student Lesson Summary and Glossary

Suppose we want to find the solution to $x – y > 5$.

Graphing technology can help us graph the solution to an inequality in two variables.

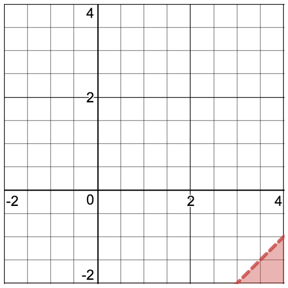
Many graphing tools allow us to enter inequalities such as $x – y > 5$ and will show the solution region, as shown here.



Some tools, however, may require the inequalities to be in slope-intercept form or another form before displaying the solution region. Be sure to learn how to use the graphing technology available in your classroom.

Although graphing using technology is efficient, we still need to analyze the graph with care. Here are some things to consider:

- The graphing window. If the graphing window is too small, we may not be able to really see the solution region or the boundary line, as shown here.



- The meaning of solution points in the situation. For example, if x and y represent the lengths of two sides of a rectangle, then only positive values of x and y (or points in the first quadrant) make sense in the situation.

Cool-down: The Band Played On (*5 minutes*)

Addressing: NC.M1.A-CED.1; NC.M1.A-REI.12

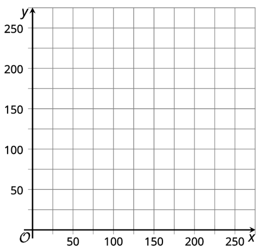
Cool-down Guidance: Points to Emphasize
Select student work from the cool-down to highlight in the next lesson, with attention to ways to determine which side of the boundary line to shade.

This cool-down requires access to graphing technology.

Cool-down

A band is playing at an auditorium with floor seats and balcony seats. The band wants to sell the floor tickets for \$15 each and balcony tickets for \$12 each. They want to make at least \$3,000 in ticket sales.

1. How much money will they collect for selling x floor tickets?
2. How much money will they collect for selling y balcony tickets?
3. Write an inequality whose solutions are the number of floor and balcony tickets sold if they make at least \$3,000 in ticket sales.
4. Use technology to graph the solutions to your inequality, and sketch the graph.



Student Reflection:

Over the last few days, the amount of time I practice is:

a. More than previously b. About the same c. Less than previously

If you're lacking practice time, what barriers keep you from doing more?

a. Time b. Lack of help c. Lack of confidence

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What unfinished learning or misunderstandings do your students have about two-variable inequalities? How did you leverage those misconceptions in a positive way to further the understanding of the class?

Practice Problems

1. This year, students in the 9th grade are collecting dimes and quarters for a school fundraiser. They are trying to collect more money than the students who were in the 9th grade last year. The students in 9th grade last year collected $143.88.

   Using d to represent the number of dimes collected and q to represent the number of quarters, which statement **best** represents this situation?

   a. $0.25d + 0.1q \geq 143.88$
   b. $0.25q + 0.1d \geq 143.88$
   c. $0.25d + 0.1q > 143.88$
   d. $0.25q + 0.1d > 143.88$

2. A farmer is creating a budget for planting soybeans and wheat. Planting soybeans costs $200 per acre, and planting wheat costs $500 per acre. He wants to spend no more than $100,000 planting soybeans and wheat.

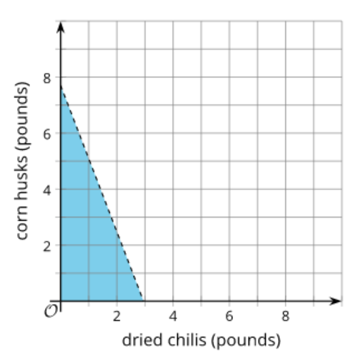
   a. Write an inequality to describe the constraints. Specify what each variable represents.

   b. Name one solution to the inequality and explain what it represents in that situation.

3. Priya is ordering dried chili peppers and corn husks for her cooking class. Chili peppers cost $16.95 per pound, and corn husks cost $6.49 per pound.

   Priya spends less than $50 on d pounds of dried chili peppers and h pounds of corn husks.

   Here is a graph that represents this situation.

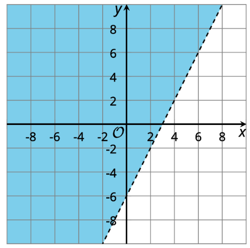


   a. Write an inequality that represents this situation.

   b. Can Priya purchase 2 pounds of dried chili peppers and 4 pounds of corn husks and spend less than $50? Explain your reasoning.

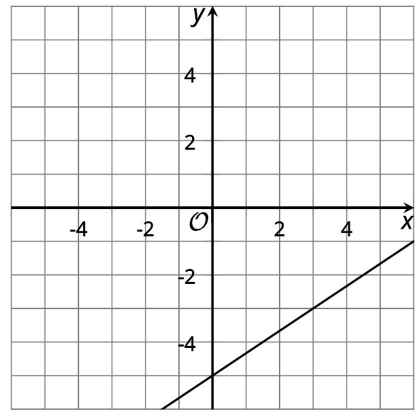
   c. Can Priya purchase 1.5 pounds of dried chili peppers and 3 pounds of corn husks and spend less than $50? Explain your reasoning.

4. Which inequality is represented by the graph?



   a. $4x - 2y > 12$
   b. $4x - 2y < 12$
   c. $4x + 2y > 12$
   d. $4x + 2y < 12$

5. Here is a graph of the equation: $2x - 3y = 15$.



   a. Are the points $(1.5, -4)$ and $(4, -4)$ solutions to the equation? Explain or show how you know.

   b. Check if each of these points is a solution to the inequality $2x - 3y < 15$:

      - $(0, -5)$
      - $(4, -2)$
      - $(2, -4)$
      - $(5, -1)$

   c. Shade the solutions to the inequality.

   d. Are the points on the line included in the solution region? Explain how you know.

   (From Unit 3, Lesson 16)

6. A store sells notepads in packages of 24 and packages of 6. The organizers of a conference need to prepare at least 200 notepads for the event.

   a. Would they have enough notepads if they bought these quantities?

      i. Seven packages of 24 and one package of 6

      ii. Five packages of 24 and fifteen packages of 6

   b. Write an inequality to represent the relationship between the number of large and small packages of notepads and the number of notepads needed for the event.

   c. Use graphing technology to graph the solution set to the inequality. Then, use the graph to name two other possible combinations of large and small packages of notepads that will meet the number of notepads needed for the event.

   (From Unit 3, Lesson 15)

7. Triangle ABC has vertices $A\,(6, 3)$, $B\,(1, 7)$, and $C\,(11, -1)$. Is triangle ABC equilateral, isosceles, or scalene? How do you know?

   (From Unit 3, Lessons 13 & 14)

8. Elena is solving this system of equations: $\begin{cases} 10x - 6y = 16 \\ 5x - 3y = 8 \end{cases}$

   She multiplies the second equation by 2, then subtracts the resulting equation from the first. To her surprise, she gets the equation $0 = 0$.

   What is special about this system of equations? Why does she get this result and what does it mean about the solutions? (If you are not sure, try graphing them.)

   (From Unit 3, Lesson 12)

9. Jada has a sleeping bag that is rated for 30° F. This means that if the temperature outside is at least 30° F, Jada will be able to stay warm in her sleeping bag.

    a. Write an inequality that represents the outdoor temperature at which Jada will be able to stay warm in her sleeping bag.

    b. Write an inequality that represents the outdoor temperature at which a thicker or warmer sleeping bag would be needed to keep Jada warm.

    (From Unit 2)

10. What is the solution set to this inequality: $6x-20 > 3(2-x) + 6x-2$?

    (From Unit 2)

11. The school band director determined from past experience that if they charge t dollars for a ticket to the concert, they can expect attendance of $1000 - 50t$. The director used this model to figure out that the ticket price needs to be $8 greater in order for at least 600 to attend. Do you agree with this claim? Why or why not?[2]

    (Addressing NC.7.EE.4b)

Lesson 18: Solutions to Systems of Linear Inequalities in Two Variables

PREPARATION

Lesson Goals	Learning Targets
• Given descriptions and graphs that represent two constraints, identify values that satisfy each constraint and those that satisfy both constraints simultaneously. • Understand that the solution set to a system of inequalities in two variables is comprised of all pairs of values that make both inequalities true, and that it is represented graphically by the region where the graphs overlap. • Write systems of inequalities in two variables, use technology to graph the solutions, and interpret the solutions in context.	• When given descriptions and graphs that represent two different constraints, I can find values that satisfy each constraint individually and values that satisfy both constraints at once. • I know what is meant by "the solutions to a system of inequalities" and can describe the graphs that represent the solutions. • I can write a system of inequalities to describe a situation, find the solutions by graphing, and interpret points in the solution region.

Lesson Narrative

Earlier in the unit, students solved systems of linear equations in two variables. They also found solutions to linear inequalities in two variables. In this lesson, students build on those understandings to find the solutions to systems of linear inequalities in two variables.

Students learn that two linear inequalities that represent the constraints in the same situation form a **system of inequalities**, and that the **solutions to the system** include all numbers that satisfy both constraints simultaneously. Graphically, the solution set to the system is represented by the region where the graphs of the individual inequalities overlap.

Throughout the lesson, students make sense of problems and persevere in solving them (MP1), and reason quantitatively and abstractly (MP2) to find values that satisfy multiple constraints, both in terms of a situation and in the absence of one.

What strategies or representations do you anticipate students might use in this lesson?

Focus and Coherence

Building On	Addressing
NC.M1.A-REI.6: Use tables, graphs, or algebraic methods (substitution and elimination) to find approximate or exact solutions to systems of linear equations and interpret solutions in terms of a context.	**NC.M1.A-CED.3:** Create systems of linear equations and inequalities to model situations in context. **NC.M1.A-REI.12:** Represent the solutions of a linear inequality or a system of linear inequalities graphically as a region of the plane.

Agenda, Materials, and Preparation

Graphing technology is needed in this lesson. Acquire devices that can run Desmos (recommended) or other graphing technology. It is ideal if each student has their own device.

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
- **Activity 3** (*Optional, 15 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U3.L18 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

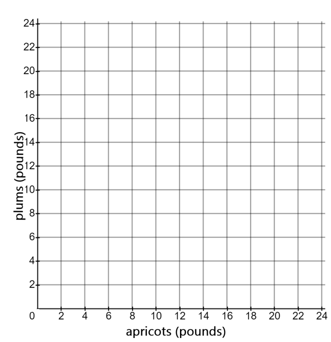
Building On: NC.M1.A.CED.3

This bridge is to help students connect writing an equation and explaining possible solutions to a contextual situation with one constraint: the total pounds of fruit picked.

Student Task Statement

Jada goes to an orchard to pick plums and apricots to make jam. She picks 20 pounds of fruit altogether.

1. Write an equation that represents the situation. Then graph the equation.
2. If Jada picks a pound of apricots, how many pounds of plums does she pick?
3. Does the point $(5, 16)$ represent a combination of pounds of plums and pounds of apricots that satisfies the constraints in this situation? Explain your reasoning.



DO THE MATH	PLANNING NOTES

Warm-up: A Silly Riddle (*5 minutes*)

Instructional Routine: Graph It
Building On: NC.M1.A-REI.6

This *Graph It* warm-up reminds students about systems of equations and their solutions. Students recall that a solution to a linear equation in two variables is any pair of numbers that makes the equation true, and that a solution to a system of two equations in two variables is a pair of numbers that make both equations true.

The given system has a solution that may be found mentally but can be calculated algebraically or by using graphing technology.

Step 1

- Give students 2 minutes to work individually.

Student Task Statement

Here is a riddle: "I am thinking of two numbers that add up to 20. The difference between them is 6. What are the two numbers?"

1. Name any pair of numbers whose sum is 20.
2. Name any pair of numbers whose difference is 6.
3. The riddle can be represented with two equations. Write the equations.
4. Solve the riddle. Explain or show your reasoning.

Step 2

- Ask students to share their responses and briefly describe their solving process.
- Record and display their reasoning, making sure to include graphical reasoning.
- Highlight that the riddle can be solved by writing and solving a system of equations. Each equation represents a constraint. Ask students:
  - "What constraints do the two equations represent?" (The first equation represents a constraint about the sum of the two numbers. The second represents a constraint about the difference of the two numbers.)
  - "What does a solution to the system represent?" (The solution is a pair of numbers that simultaneously meet both constraints or make both equations true.)
  - "How many pairs of numbers meet both constraints at the same time?" (Only one pair. The graphs of the equations intersect at one point.)

DO THE MATH	PLANNING NOTES

Activity 1: A Quilting Project (*15 minutes*)

Instructional Routine: Graph It
Addressing: NC.M1.A-CED.3; NC.M1.A-REI.12

In this *Graph It* activity, students encounter a situation in which two constraints that can be expressed with inequalities are represented on two separate graphs, which makes it challenging to find pairs of values that meet both constraints simultaneously. This motivates a desire to represent both constraints on the same graph.

Making graphing technology available gives students an opportunity to choose appropriate tools strategically (MP5).

Step 1

- Give students a moment to read the task statement and look at the two graphs. Ask students:
  - "What are the two constraints in this situation?" (a length constraint and a cost constraint)
  - "Which graph represents which constraint? How do you know?" (The first graph represents the length constraint. Possible explanations:
    - The graph intersects the vertical and horizontal axes at approximately 9.5, which means that if the quilter bought 0 yards of one color, he will need at least 9.5 yards of the other color.
    - The length constraint says "at least 9.5 yards," so the lengths must include values greater than 9.5, which is shown by the shaded region of the first graph.
    - The second graph represents the cost constraint. If the quilter bought 0 yards of the light color, he could buy up to $\frac{110}{13}$, or about 8.5, yards of the dark color fabric. This corresponds to the vertical intercept of the second graph.
    - The cost constraint says "up to $110," so the lengths must be below certain limits. The solution region of the second graph shows values below a boundary.)
- Ask students to arrange themselves with a partner to work on the task or use visibly random grouping. Encourage students to each take two points in questions 2 and 4, then discuss any patterns noticed in relationship to the structure of the inequalities.

RESPONSIVE STRATEGIES

Encourage and support opportunities for peer interactions. During the time spent on the two questions in the launch, invite students to brainstorm with a partner before sharing with the whole-class. Display sentence frames that elicit descriptive observations: "I notice that. . .", as well as frames that support interpretation and representation "____ represents ____ because . . . ".

Supports accessibility for: Language; Social-emotional skills

Monitoring Tip: As students work, monitor for the different ways students try to find a pair of values that satisfy both inequalities. Some likely approaches:

- Substituting different x- and y-values into the inequalities until they find a pair that make both inequalities true.
- Visually estimating a point in the solution region of one graph that appears likely to also be in the solution region of the other graph (for example, noticing that $(6,4)$ is in the solution region of each inequality.
- Graphing both inequalities on the same coordinate plane (either using technology or by copying the line in one graph onto the other graph) and finding a point in the region where they overlap.

Identify students who use these or other approaches and let them know that they may be asked to share during class discussion.

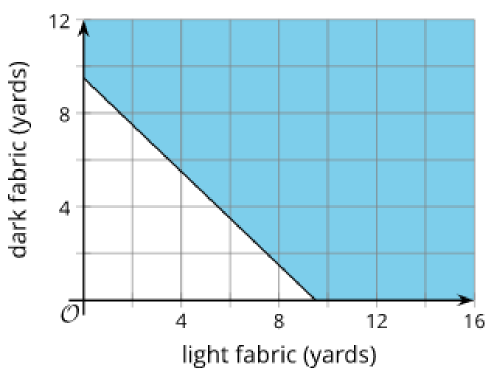
Student Task Statement

To make a quilt, a quilter is buying fabric in two colors, light and dark. He needs at least 9.5 yards of fabric in total.

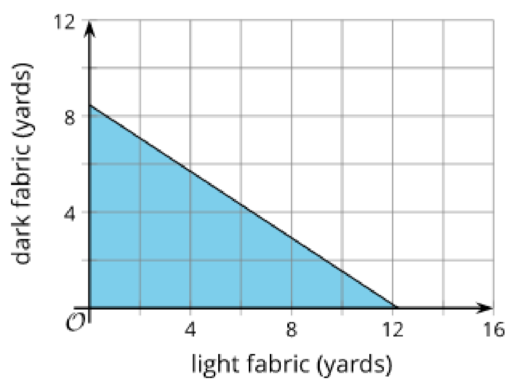
The light color costs $9 a yard. The dark color costs $13 a yard. The quilter can spend up to $110 on fabric.

Here are two graphs that represent the two constraints.

Graph A



Graph B



1. Write an inequality to represent the length constraint. Let x represent the yards of light fabric and y represent the yards of dark fabric.

2. Select **all** the pairs that satisfy the length constraint.

   a. $(5,5)$
   b. $(2.5,4.5)$
   c. $(7.5,3.5)$
   d. $(12,10)$

3. Write an inequality to represent the cost constraint.

4. Select **all** the pairs that satisfy the cost constraint.

   a. $(1,1)$
   b. $(4,5)$
   c. $(8,3)$
   d. $(10,1)$

5. Explain why $(2,2)$ satisfies the cost constraint, but not the length constraint.

6. Find at least one pair of numbers that satisfies **both** constraints. Be prepared to explain how you know.

7. What does the pair of numbers represent in this situation?

Step 2

- Focus the discussion on the last two questions: *Find at least one pair of numbers that satisfies both constraints* and *what does this pair of numbers represent in this situation?*
- Select previously identified students to share how they identified a pair of values that meet both constraints in this order (from Monitoring Tip):
  - First: A student that substituted different x and y values into the inequalities until they found a pair that made both inequalities true.
  - Second: A student who visually estimated a point in the solution region of one graph that appears to be in the solution region of the other graph.
  - Third: A student who graphed both inequalities on the same coordinate plane. Select students who graphed both inequalities on the same plane to display their graphs for all to see.
- If no students did this on their own, display the Desmos applet for all to see.
- Discuss the meaning of a pair of values that satisfy both inequalities. Emphasize that, in this situation, it refers to the amount of fabric of each color that meets both the length and cost requirements.
- Explain to students that the two inequalities representing the constraints in the same situation form a system of linear inequalities. In a system of linear equations, the solutions can be represented by one or more points where the graphs of the equations intersect. The solutions to a system of inequalities are all points in the region where the graphs of the two inequalities overlap, because those points represent all pairs of values that make both inequalities true.

DO THE MATH	PLANNING NOTES

Activity 2: Remember These Situations? (*10 minutes*)

Instructional Routines: Graph It; Discussion Supports (MLR8); Stronger and Clearer Each Time (MLR1) - Responsive Strategy
Addressing: NC.M1. A-CED.3; NC.M1.A-REI.12

In this *Graph It* activity, students write systems of inequalities that represent situations and find the solutions by graphing. Students have encountered the same situations and constraints in Lesson 16, Activity 1, so the main work here is on thinking about each pair of constraints as a system, finding the solution region of each inequality, and identifying a point in the region where the graphs overlap as a solution to the system.

Step 1

- Ask students to arrange themselves in groups of three or use visibly random grouping.
- Explain to students that they will now revisit some situations they have seen and graphed in an earlier lesson.
- Assign one situation (Bank Accounts, Concert Tickets, or Advertising Packages) to each group member (or allow them to choose one situation). If students worked with only one situation in Lesson 16, that would be the ideal situation to continue with.
- Give students 3 minutes of individual quiet work time to complete their situation.
- Use *Discussion Supports* to support small-group discussion by giving students 1 minute to plan what they will say when they present their responses and graph to their group.
  - Circulate and support students to identify what details are important to share, and to think about how they will explain their reasoning using mathematical language they have heard and used over the course of this unit.
- Ask students to present their responses and graph to their group.

RESPONSIVE STRATEGY

To support development of organizational skills, check in with students within the first 2–3 minutes of work time. Look for students who are organizing their information, understanding the need for two inequalities, and strategizing as to how to overlay them onto a single graph.

Supports accessibility for: Memory; Organization

Advancing Student Thinking: Students may need reminding what "a solution to the system" would be in these specific contexts. (First find a point where the total money in the two accounts totals less than or equal to $600. Now make sure the money in the savings account is also at least $50.)

Student Task Statement

Here are some situations you have seen before. Answer the questions for one situation.

Bank Accounts

- A customer opens a checking account and a savings account at a bank. They will deposit a maximum of $600, some in the checking account and some in the savings account. (They might not deposit all of it and instead keep some of the money as cash.)
- The bank requires a minimum balance of $50 in the savings account. It does not matter how much money is kept in the checking account.

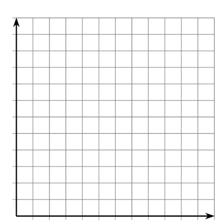
Concert Tickets

- Two kinds of tickets to an outdoor concert were sold: lawn tickets and seat tickets. Fewer than 400 tickets in total were sold.
- Lawn tickets cost $30 each and seat tickets cost $50 each. The organizers want to make at least $14,000 from ticket sales.

Advertising Packages

- An advertising agency offers two packages for small businesses who need advertising services. A basic package includes only design services. A premium package includes design and promotion. The agency's goal is to sell at least 60 packages in total.
- The basic advertising package has a value of $1,000 and the premium package has a value of $2,500. The goal of the agency is to sell more than $60,000 worth of small-business advertising packages.

1. Write a system of inequalities to represent the constraints. Specify what each variable represents.
2. Use technology to graph the inequalities and sketch the solution regions. Include labels and scales for the axes.
3. Identify a solution to the system. Explain what the numbers mean in the situation.



Step 2

- Facilitate a whole-class discussion to emphasize the meaning of a point in the region where two graphs of linear inequalities overlap. Make sure students understand that all the points in that region represent values that simultaneously meet both constraints in the situation.
  - Ask students: "Why does it make sense to think of the two inequalities in each situation as a system and find the solutions to the system, instead of only to individual inequalities?" (If both constraints in the situation must be met, then we need to find values that satisfy both inequalities.)

RESPONSIVE STRATEGY

Use the Stronger and Clearer Each Time routine to give students an opportunity to clarify the language they use to communicate about finding solutions to systems rather than individual inequalities. Give students 1 minute to jot notes about this. Give them 1 minute to share with a partner and get feedback and additional ideas and words/phrases. Have students repeat the sharing and feedback with another partner. Finally give them 1-2 minutes to write a second draft that is stronger and clearer than their initial writing.

Stronger and Clearer Each Time (MLR1)

DO THE MATH	PLANNING NOTES

Activity 3: Scavenger Hunt (*Optional, 15 minutes*)

Building On: NC.M1.A-REI.12

This optional activity reinforces the idea that the solutions to a system of inequalities can be effectively represented by a region on the graphs of the inequalities in the system. The activity is designed to be completed without the use of graphing technology.

Step 1

- Ask students to select a partner or use visibly random grouping.
- Launch the task by reviewing the scavenger hunt and the clues to review.

Advancing Student Thinking: Some students may have trouble interpreting the graph of the fourth system, wondering if a point in either of the shaded regions on the graph could be where an item is hidden. Ask them to pick a point on the graph and consider whether it satisfies the first inequality, and then whether it satisfies the second inequality. Remind them that a solution to a system needs to satisfy both.

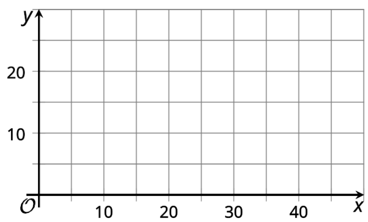
Student Task Statement

Members of a high school math club are doing a scavenger hunt. Three items are hidden in the park, which is a rectangle that measures 50 meters by 20 meters.

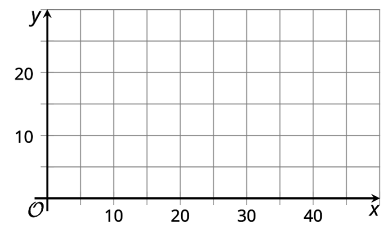
- The clues are written as systems of inequalities. One system has no solutions.
- The locations of the items can be narrowed down by solving the systems. A coordinate plane can be used to describe the solutions.

Can you find the hidden items? Sketch a graph to show where each item could be hidden.

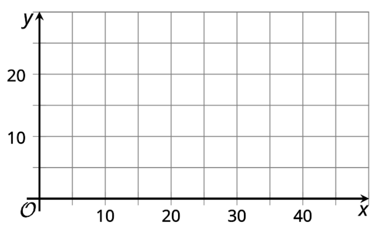
Clue 1: $\begin{cases} y > 14 \\ x < 10 \end{cases}$



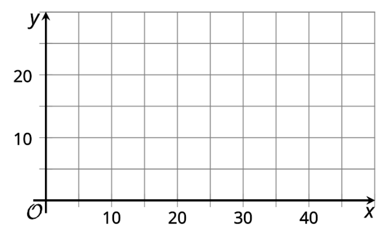
Clue 2: $\begin{cases} x + y < 20 \\ x > 6 \end{cases}$



Clue 3: $\begin{cases} y < -2x + 20 \\ y < -2x + 10 \end{cases}$



Clue 4: $\begin{cases} y \geq x + 10 \\ x > y \end{cases}$



Are You Ready For More?

Two non-negative numbers x and y satisfy $x + y \leq 1$.

1. Find a second inequality, also using x and y values greater than or equal to zero, to make a system of inequalities with exactly one solution.
2. Find as many ways to answer this question as you can.

Step 2

- Invite students to share their graphs and strategies for finding the solution regions. In particular, discuss how they found out which system had no solutions.
- Remind students that a system of linear equations has no solutions if the graphs of the equations are two parallel lines that never intersect. Explain that a system of linear inequalities has no solutions if their regions are bound by two parallel lines and the solution region of each one is on the "outside" of the parallel lines, as is the case with the last given system.

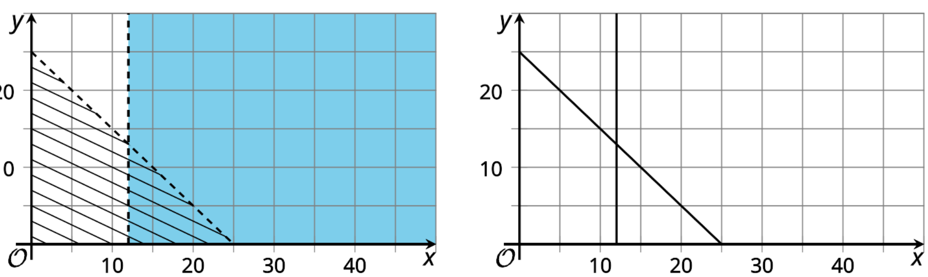
DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is to introduce students to systems of inequalities. Like systems of equations, systems of inequalities are used to represent situations involving multiple constraints. In this debrief, students will review what they learned in this lesson by comparing and contrasting the two.

To help students make connections between systems of equations and systems of inequalities, display the following graphs for all to see.



Ask students:

- "How are the two sets of graphs alike?" (They have the same two lines. They can tell us about the solutions to individual equations or inequalities, as well as the solutions to systems.)
- "How are they different?" (The first set of graphs show two regions that overlap, bounded by dotted lines. The second set shows two intersecting lines and the lines are solid. One set represents the solutions to a system of linear inequalities.)
- "How can we tell the number of solutions from each set of graphs?" (The graphs representing a system of equations show one point of intersection, so there is only one solution. The graphs representing a system of inequalities show one region of overlap, but there are many points in that region. This means that there are many solutions.)

PLANNING NOTES

Student Lesson Summary and Glossary

In this lesson, we used two linear inequalities in two variables to represent the constraints in a situation. Each pair of inequalities forms a **system of inequalities**.

System of inequalities: Two or more inequalities that represent the constraints in the same situation.

A **solution to the system** is any (x, y) pair that makes both inequalities true, or any pair of values that simultaneously meet both constraints in the situation. The set of all solutions to the system is often best represented by a region on a graph.

Solutions to a system of inequalities: All pairs of values that make the inequalities in a system true. The solutions to a system of inequalities can be represented by the points in the region where the graphs of the two inequalities overlap. We also call these solutions the **solution set**.

Suppose there are two numbers, x and y, and there are two things we know about them:

- The value of one number is more than double the value of the other.
- The sum of the two numbers is less than 10.

We can represent these constraints with a system of inequalities.

$$\begin{cases} y > 2x \\ x + y < 10 \end{cases}$$

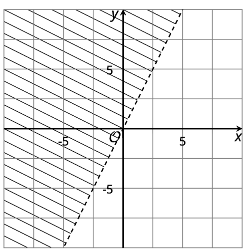
There are many possible pairs of numbers that meet the first constraint, for example: 1 and 3, or 4 and 9.

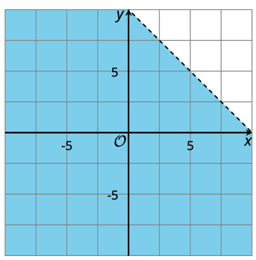
The same can be said about the second constraint, for example: 1 and 3, or 2.4 and 7.5.

The pair $x = 1$ and $y = 3$ meets both constraints, so it is a solution to the system.

The pair $x = 4$ and $y = 9$ meets the first constraint but not the second ($9 > 2(4)$ is a true statement, but $4 + 9 < 10$ is not true.)

Remember that graphing is a great way to show all the possible solutions to an inequality, so let's graph the solution region for each inequality.

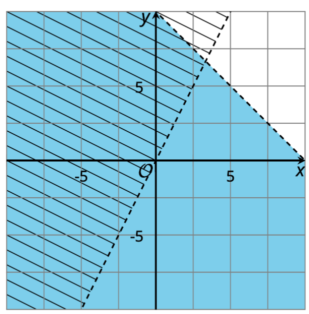




Because we are looking for a pair of numbers that meet both constraints or make both inequalities true at the same time, we want to find points that are in the solution regions of both graphs.

To do that, we can graph both inequalities on the same coordinate plane.

The solution set to the system of inequalities is represented by the region where the two graphs overlap.



Cool-down: Oh Good, Another Riddle (*5 minutes*)

Addressing: NC.M1.A-CED.3; NC.M1.A-REI.12

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next lessons to support students in advancing their current understanding.

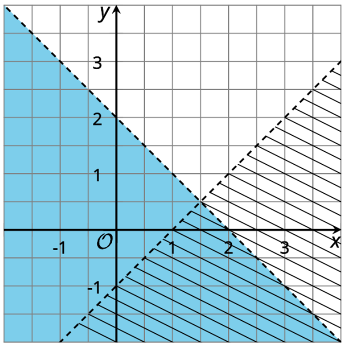
Cool-down

Here is another riddle:

- The sum of two numbers is less than 2.
- If we subtract the second number from the first, the difference is greater than 1.

What are the two numbers?

1. The riddle can be represented by a system of inequalities. Write an inequality for each statement.
2. These graphs represent the inequalities in the system. Which graph represents which inequality?
3. Name a possible solution to the riddle. Explain or show how you know.



Student Reflection:

After today's lesson, how confident do you feel about combining what you know about systems of equations and graphing inequalities? Feel free to share details on how you are feeling.

a. Very confident b. Somewhat confident c. Not confident

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

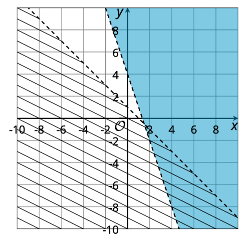
What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

As students worked in their small groups today, whose ideas were heard, valued, and accepted? How can you adjust the group structure tomorrow to ensure each student's ideas are a part of the collective learning?

Practice Problems

1. Two inequalities are graphed on the same coordinate plane.

   Which region represents the solution to the system of the two inequalities?



2. Select **all** the pairs of x and y that are solutions to the system of inequalities: $\begin{cases} y \leq -2x + 6 \\ x - y < 6 \end{cases}$

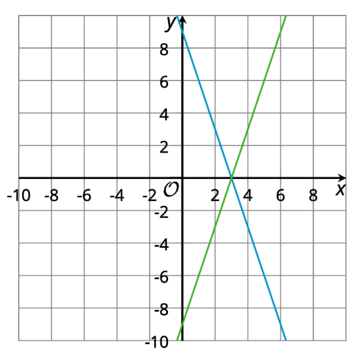
   a. $x = 0, y = 0$
   b. $x = -5, y = -15$
   c. $x = 4, y = -2$
   d. $x = 3, y = 0$
   e. $x = 10, y = 0$

3. Jada has $200 to spend on flowers for a school celebration. She decides that the only flowers that she wants to buy are roses and carnations. Roses cost $1.45 each and carnations cost $0.65 each. Jada buys enough roses so that each of the 75 people attending the event can take home at least one rose.

   a. Write an inequality to represent the constraint that every person takes home at least one rose.

   b. Write an inequality to represent the cost constraint.

4. Here are the graphs of the equations $3x + y = 9$ and $3x - y = 9$ on the same coordinate plane.



   a. Label each graph with the equation it represents.

   b. Identify the region that represents the solution set to $3x + y < 9$. Is the boundary line a part of the solution? Use a colored pencil or cross-hatching to shade the region.

   c. Identify the region that represents the solution set to $3x - y < 9$. Is the boundary line a part of the solution? Use a different colored pencil or cross-hatching to shade the region.

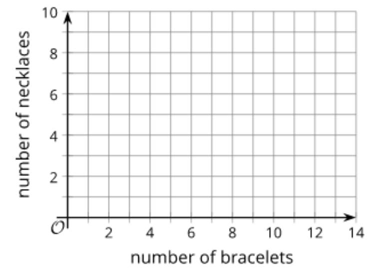
   d. Identify a point that is a solution to both $3x + y < 9$ and $3x - y < 9$.

5. In physical education class, Mai takes 10 free throws and 10 jump shots. She earns 1 point for each free throw she makes and 2 points for each jump shot she makes. The greatest number of points that she can earn is 30.

   a. Write an inequality to describe the constraints. Specify what each variable represents.

   b. Name one solution to the inequality and explain what it represents in that situation.

   (From Unit 3, Lesson 17)



6. A rectangle with a width of w and a length of l has a perimeter greater than 100.

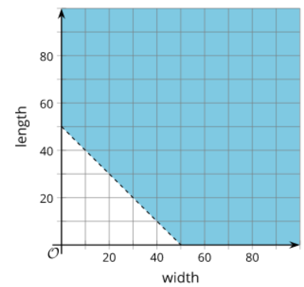
   Here is a graph that represents this situation.

   a. Write an inequality that represents this situation.

   b. Can the rectangle have a width of 45 and a length of 10? Explain your reasoning.

   c. Can the rectangle have a width of 30 and a length of 20? Explain your reasoning.

   (From Unit 3, Lesson 17)



7. Which coordinate pair is a solution to the inequality $4x - 2y < 22$?

   a. $(4, -3)$
   b. $(4, 3)$
   c. $(8, -3)$
   d. $(8, 3)$

   (From Unit 3, Lesson 16)

8. Elena is considering buying bracelets and necklaces as gifts for her friends. Bracelets cost \$3, and necklaces cost \$5. She can spend no more than \$30 on the gifts.

   a. Write an inequality to represent the number of bracelets,

   b. b, and the number of necklaces n, she could buy while sticking to her budget.

   c. Graph the solutions to the inequality on the coordinate plane.

   d. Explain how we could check if the boundary is included or excluded from the solution set.

   (From Unit 3, Lesson 15)

9. Point R is at $(-3, 6)$. Find the distance from point R to:

   a. The origin

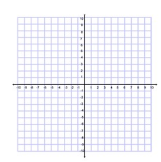
   b. The x-axis

   c. The y-axis

   d. Point $(1, -2)$

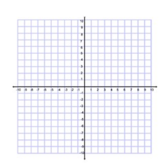
   e. Which distance was the farthest? How could you have predicted that using the coordinates?

   (From Unit 3, Lessons 13 & 14)



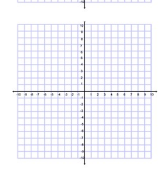
10. The perpendicular bisector of a line segment is a line perpendicular to the segment that passes through its midpoint. What is an equation for the perpendicular bisector of segment XY, with point X $(2, -6)$ and Y $(5, -5)$?

    (From Unit 3, Lessons 6 and 13 & 14)



11. A triangle has vertices $D(1,5)$, $O(7,9)$, and $G(7,2)$. Is DOG equilateral, isosceles, or neither? How do you know?

    (From Unit 3, Lesson 7)



Lesson 19: Solving Problems with Systems of Linear Inequalities in Two Variables

PREPARATION

Lesson Goals	Learning Target
• Given a system of inequalities and their graphs, explain (orally and in writing) how to tell if a pair of values is a solution to the system. • Practice writing systems of inequalities in two variables and finding the solution sets by reasoning or by graphing.	• I can explain how to tell if a point on the boundary of the graph of the solutions to a system of inequalities is a solution or not.

Lesson Narrative

In a previous lesson, students learned that the solutions to a system of linear inequalities can be represented graphically with overlapping regions.

In this lesson, students take a closer look at whether points on the boundary lines of the system's solution region are included in the solutions. Analyzing graphs and communicating observations about them require attention to precision (MP6). Students also apply these insights to solve more challenging contextual problems. This work involves making sense of the information needed to solve the problems (MP1).

Where do you see connections from what students shared and discussed in previous lessons to this lesson?

Focus and Coherence

Building On	Addressing
NC.8.EE.8: Analyze and solve a system of two linear equations in two variables in slope-intercept form. • Understand that solutions to a system of two linear equations correspond to the points of intersection of their graphs because the point of intersection satisfies both equations simultaneously. • Solve real-world and mathematical problems leading to systems of linear equations by graphing the equations. Solve simple cases by inspection.	**NC.M1.A-CED.3:** Create systems of linear equations and inequalities to model situations in context. **NC.M1.A-REI.12:** Represent the solutions of a linear inequality or a system of linear inequalities graphically as a region of the plane.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*10 minutes*)
- **Activity 2** (*15 minutes*)
  - Technology is required for this activity: Acquire devices that can access Desmos (recommended) or other graphing technology. It is ideal if each student has their own device. Be prepared to display a graph using technology for all to see.
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U3.L19 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.8.EE.8

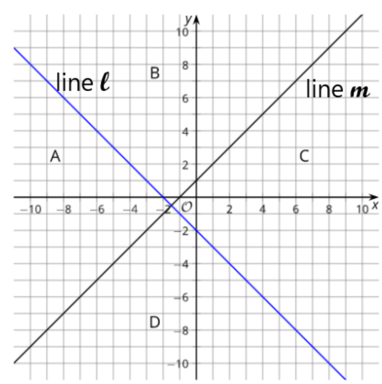
This bridge gives students an opportunity to practice interpreting regions of the plane created by two lines dividing the plane. Students select a point within a region and determine which system the point is a solution to. In the lesson, students explore the idea of a system of inequalities. Regions that are solutions to both inequalities represent the solution set to a system. This task is aligned to question 5 in Check Your Readiness.

Student Task Statement

1. The graph shows the lines $y = x + 1$ and $y = -x - 2$. Which line represents $y = x + 1$?
2. Write a coordinate pair for a point in region A.
3. For which of the following systems is the point a solution? Explain how you know.

a. $\begin{cases} y \geq x + 1 \\ y \leq -x - 2 \end{cases}$

b. $\begin{cases} y \geq x + 1 \\ y \geq -x - 2 \end{cases}$



DO THE MATH

PLANNING NOTES

Warm-up: Graphs of Solutions *(5 minutes)*

Instructional Routine: Which One Doesn't Belong?	
Building Towards: NC.M1.A-REI.12	

This warm-up prompts students to carefully analyze and compare graphs that represent linear equations and inequalities. Making comparisons prompts students to think about the solutions to the equations, inequalities, or systems that are being represented. It also gives students a reason to use language precisely (MP6).

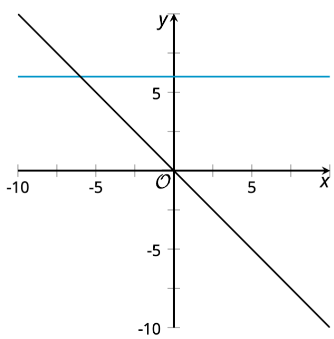
Step 1

- Ask students to arrange themselves in small groups or use visibly random grouping.
- Display the graphs for all to see.
- Ask students to indicate when they have noticed one that does not belong and can explain why.
- Then, use the *Which One Doesn't Belong?* routine. Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, tell each student to share their reasoning why a particular item does not belong and together find at least one reason each item doesn't belong.

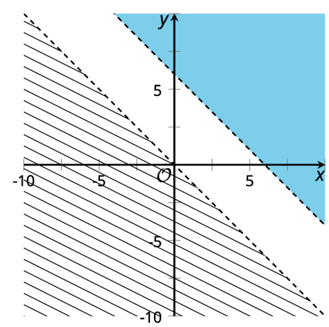
Student Task Statement

Which one doesn't belong? Explain your reasoning.

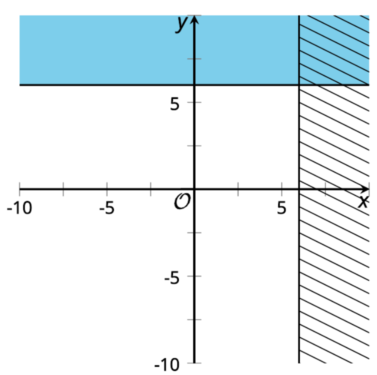
a.



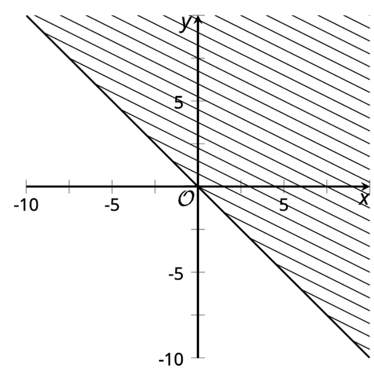
b.



c.



d.



Step 2

- Ask each group to share one reason why a particular item does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given are correct.
- During the discussion, ask students to explain the meaning of any terminology they use, such as "infinitely many solutions" or "boundary line." Also, press students on unsubstantiated claims.

DO THE MATH	PLANNING NOTES

Activity 1: Focusing on the Details *(10 minutes)*

Instructional Routines: Take Turns; Discussion Supports (MLR8) - Responsive Strategy	
Addressing: NC.M1.A-REI.12	

Previously, students have learned that any point that is in the overlapping solution regions of the graphs of two inequalities is a solution to the system formed by those inequalities. In this activity, students take a closer look at whether points that are on the boundary lines are solutions to the system.

Step 1

- Ask students to arrange themselves into pairs or use visibly random grouping.
- Display the system of inequalities and the graphs for all to see.
- Give students a minute of quiet time to think about which region represents the solutions to each inequality and be prepared to explain how they know. Then, give students another minute to discuss their thinking with a partner.
- Facilitate a whole-class discussion.
  - Students are likely to identify the inequality that each graph represents by considering the equation of the boundary line. They may relate the solidly shaded region to $x < y$ because the dashed line is the graph of $x = y$, or they may relate the hashed region to the solutions of $y \geq -2x-6$ because the boundary line has a negative slope and it intersects the y-axis at $(0, -6)$.
  - Other students may test some coordinate pairs in each region and see if they make an inequality true. For example, they may say that all points above the graph of $x = y$ have an x-value that is less than the y-value.

RESPONSIVE STRATEGIES

Prior to independent work, engage the whole-class in developing a set of directions that displays criteria for checking if points are a solution. This can be written as a flow chart or as a list. Recommend students start with the step of plotting the point on their graph. Support them in articulating criteria that address evaluating points in shaded regions and on boundary lines. Check for understanding by inviting students to rephrase directions in their own words. Consider keeping the display of directions visible throughout the activity.

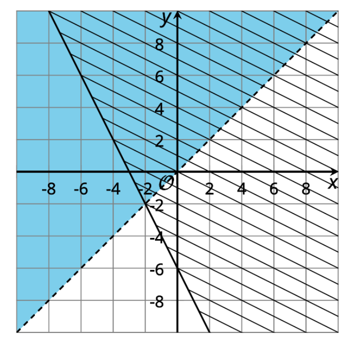
Supports accessibility for: Language; Memory

– If these strategies for connecting the algebraic and graphical representations are not mentioned by students, bring them up.

Tell students that they will use the *Take Turns* routine with a partner to determine whether certain points on the coordinate plane are solutions to the system. Remind students this means that as they work with their partner, they will be explaining, justifying, agreeing or disagreeing, and asking clarifying questions. If they disagree, they are expected to support their case and listen to their partner's arguments.

Student Task Statement

Here are the graphs of the inequalities in this system: $\begin{cases} x < y \\ y \geq -2x - 6 \end{cases}$

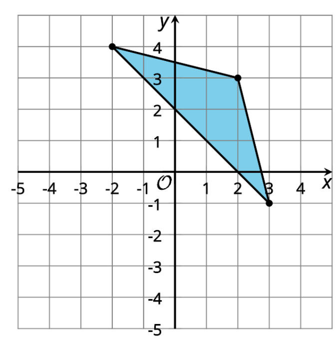


Decide whether each point is a solution to the system. Be prepared to explain how you know.

1. $(3, -5)$
2. $(0, 5)$
3. $(-6, 6)$
4. $(3, 3)$
5. $(-2, -2)$

Are You Ready For More?

Find a system of inequalities with this triangle as its set of solutions.



Step 2

- Facilitate a whole-class discussion. Focus the discussion on the points on the boundary lines and how students determine if they are or are not solutions to the system.
- Highlight explanations that state that a solution to a system of linear inequalities must be a solution to every inequality in the system. If a point on the boundary line is not included in the solution set to one inequality (so the graph is a dashed line), then it is also not included in the solution set to the system.

RESPONSIVE STRATEGY

Use this routine to support whole-class discussion. For each observation that is shared, ask students to restate what they heard using precise mathematical language. Consider providing students time to restate what they hear to a partner before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped clarify the original statement. This provides more students with an opportunity to produce language as they interpret the reasoning of others.

Discussion Supports (MLR8)

DO THE MATH	PLANNING NOTES

Activity 2: Pet Sitters *(15 minutes)*

Instructional Routine: Three Reads (MLR6)
Addressing: NC.M1.A-CED.3; NC.M1.A-REI.12

In this activity, students represent a contextual situation by creating a system of linear inequalities, graphing the solution set and interpreting solutions.

Step 1

- Ask students to arrange themselves into pairs or use visibly random grouping.

Use the *Three Reads* routine to support comprehension of this word problem. This routine helps students interpret the language within a given situation needed to create a system of linear inequalities.

- First Read: Without displaying the task, read the context aloud to the class: "Andre and Elena are starting a pet sitting business to care for cats and dogs while their owners are on vacation. Space: Cat pens will require 6 square feet of space, while dog runs require 24 square feet. Andre and Elena have up to 360 square feet available in the storage shed for pens and runs, while still leaving enough room to move around the cages. Startup Costs: Andre and Elena plan to invest much of the $1280 they earned from their last business venture to purchase cat pens and dog runs. It will cost $32 for each cat pen and $80 for each dog run."
    - Ask students: "What is this situation about? What is going on here?"
    - Let students know the focus is just on the situation, not on the numbers (for example, students might say "this is about a pet sitting business where there are cat pens and dog runs" or "investing in a pet sitting business with different costs").
    - Clarify any unfamiliar words (e.g., "pen" and "run" in this context). Visuals often help.
- Second Read: Display the description of the situation (without the problems) and ask a student volunteer to read it aloud to the class again.
    - Ask: "What are the quantities in this situation? A quantity is something that can be counted or measured."
    - Spend less than a minute scribing student responses.
    - Write down all ideas students generate, but listen for, and amplify (for example, by revoicing and clarifying as needed) the important quantities that vary in relation to each other in this situation: square footage of a cat pen and a dog run, cost of a cat pen and a dog run, total amount of square footage, and total amount of money.

- Third Read: Invite students to read the situation again to themselves, or ask another student volunteer to read it aloud. Then reveal the first problem (or all of the problems, if logistically simpler), and ask students to read the problem(s) to themselves.
  - Ask students: “How might you approach the problem(s)? What is the first thing you will do?”
  - Spend 1–2 minutes scribing student ideas as they brainstorm possible starting points.
  - Be sure to stop any students who begin to share a complete solution; the goal is to crowdsource some starting points.

Step 2

- Provide students 2–3 minutes of quiet time to write a system of linear inequalities.
- Ask students to discuss their thinking with their partner then continue working together on the remaining questions. For question 3, encourage students to divide up the different types of points and provide a justification to their partners.
- Provide access to graphing technology to each group.

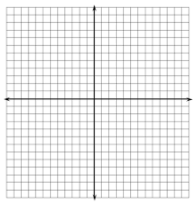
Advancing Student Thinking: Students may write a system of equations instead of inequalities. Ask students “Do Andre and Elena have to use all of the space or all of the money? If not, how do we represent all combinations that are possible, even those that don’t use all of the resources (space and money) available?”

Student Task Statement[1]

Andre and Elena are starting a pet sitting business to care for cats and dogs while their owners are on vacation.

- Space: Cat pens will require 6 square feet of space, while dog runs require 24 square feet. Andre and Elena have up to 360 square feet available in the storage shed for pens and runs, while still leaving enough room to move around the cages.
- Startup Costs: Andre and Elena plan to invest much of the $1280 they earned from their last business venture to purchase cat pens and dog runs. It will cost $32 for each cat pen and $80 for each dog run.

1. Create a system of inequalities to represent the constraints of space and startup costs.
2. Graph the system of inequalities.
3. Identify four different points and interpret what each point means in terms of the situation. Choose one point of each of these types:
   a. within the intersecting regions
   b. on one of the boundary lines
   c. on the other boundary line
   d. the intersection of the two lines



Are You Ready For More?

Andre and Elena want to make as much money as possible from their business, so they are trying to determine how many of each type of pet they should plan to accommodate. They plan to charge $8 per day for boarding each cat and $20 per day for each dog.

What combination of the number of cats and number of dogs would maximize the income? Show or explain your reasoning.

[1] Adapted from Secondary One, Module 2, Mathematics Vision project http://www.mathematicsvisionproject.org, licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0)

Step 3

- Facilitate a whole-class discussion. Focus the discussion on the location of points (on a line or within the overlapping region) and the interpretation of the solutions in context.
  - Display a graph of the system for all to see.
  - Ask students to share points they selected and their interpretation.
  - Mark the points on the graph with a dot if the point is a solution to both constraints and with an X if not a solution to both constraints.
  - Highlight combinations on each line that are solutions to both inequalities. Ask students "How does the line indicate that the points are solutions?" (The line is solid and not dotted.)

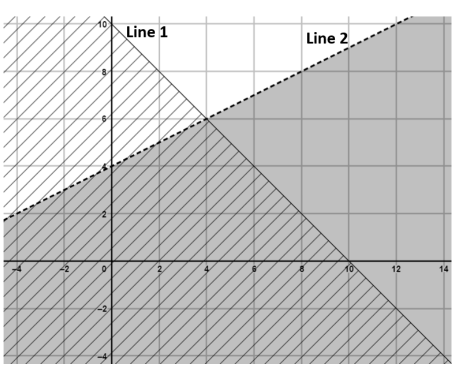
DO THE MATH	PLANNING NOTES

Lesson Debrief *(5 minutes)*

In this lesson students identify solutions to a system of linear inequalities by analyzing the graphs, paying specific attention to the boundary lines. Facilitate a discussion using the following questions.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

Display the following graph:



PLANNING NOTES

- "Are the points $(6,4)$ and $(12,-2)$ on line 1 solutions to the system? How do you know?" (Yes. These points fall within the shaded region bounded by line 2. They are also on line 1, which is a boundary line. Since line 1 is solid, the points are included in the solution set.
- "What about the point $(2,8)$?" (No, because it is not in the shaded region bounded by line 2.)
- "Are the points $(0,4)$ and $(2,5)$ on line 2 solutions to the system? How do you know?" (No. Points on a dotted line are not included.)
- "What about the point $(4,6)$?" (No. It is a solution for line 1 but not for line 2.)

Student Lesson Summary and Glossary

A family has at most $25 to spend on activities at Fun Zone. It costs $10 an hour to use the trampolines and $5 an hour to use the pool. The family can stay less than 4 hours.

What are some combinations of trampoline time and pool time that the family could choose given their constraints?

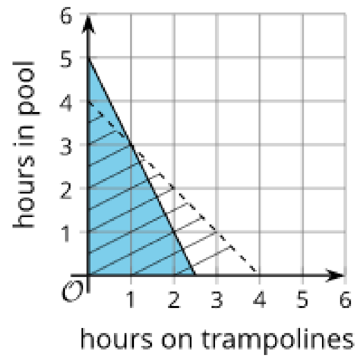
We could find some combinations by trial and error, but writing a system of inequalities and graphing the solution would allow us to see all the possible combinations.

Let t represent the time, in hours, on the trampolines and p represent the time, in hours, in the pool.

The constraints can be represented with the system of inequalities:

$$\begin{cases} 10t + 5p \leq 25 \\ t + p < 4 \end{cases}$$

Here are graphs of the inequalities in the system.



The solution set to the system is represented by the region where the shaded parts of the two graphs overlap. Any point in that region is a pair of times that meet both the time and budget constraints.

The graphs give us a complete picture of the possible solutions.

- Can the family spend 1 hour on the trampolines and 3 hours in the pool?

  No. We can reason that this is too much time because those times add up to 4 hours, and the family wants to spend *less than* 4 hours. But we can also see that the point $(1,3)$ lies on the dashed line of one graph, so it is not a solution.

- Can the family spend 2 hours on the trampolines and 1.5 hours in the pool?

  No. We know that these two times add up to less than 4 hours, but to find out the cost, we need to calculate $10(2) + 5(1.5)$, which is 27.5 and is more than the budget.

  It may be easier to see that this combination is not an option by noticing that the point $(2, 1.5)$ is in the region with line shading, but not in the region with solid shading. This means it meets one constraint but not the other.

Cool-down: Widgets and Zurls *(5 minutes)*

Addressing: NC.M1.A-REI.12

Cool-down Guidance: Press Pause
If students are still struggling to shade graphs appropriately, highlight student work from the cool-down to address misconceptions and assign practice problems from this lesson for additional opportunities for students to practice.

Graphing technology should not be used in this cool-down.

Cool-down

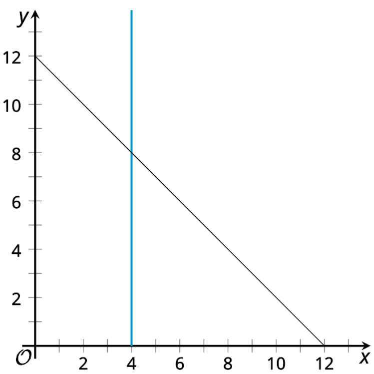
A factory produces widgets and zurls. The combined number of widgets and zurls made each day cannot be more than 12. The maximum number of widgets the factory can produce in a day is 4.

Let x be the number of widgets and y the number of zurls.

1. Select **all** the inequalities that represent this situation.
   a. $x < 4$
   b. $x \leq 4$
   c. $x > 4$
   d. $x + y > 12$
   e. $x + y \leq 12$

2. Here are graphs of $x = 4$ and $x + y = 12$.

   Complete the graphs (by shading regions and adjusting line types as needed) to show all the allowable numbers of widgets and zurls that the factory can produce in one day.



3. Does each ordered pair represent an allowable combination of widgets and zurls produced in one day?
   - $(4, 5)$
   - $(11, 1)$
   - $(4, 12)$
   - $(3, 9)$

Student Reflection: Today my participation was (*circle one*): *high* *medium* *low* because ________________________.

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Think about a recent time from class when your students were confused. What did you do to support them in reasoning about their confusion together as a community of learners?

Practice Problems

1. Jada has p pennies and n nickels that add up to more than 40 cents. She has fewer than 20 coins altogether.

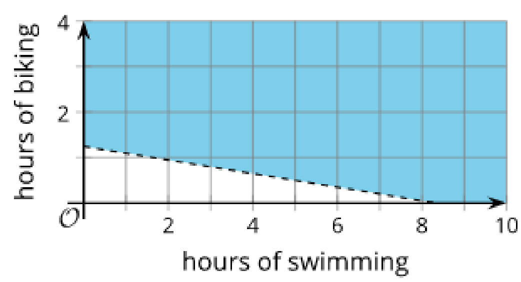
   a. Write a system of inequalities that represents how many pennies and nickels that Jada could have.

   b. Is it possible that Jada has each of the following combinations of coins? If so, explain or show how you know. If not, state which constraint(s)—the amount of money or the number of coins—it does not meet.

   - 15 pennies and 5 nickels
   - 16 pennies and 2 nickels
   - 10 pennies and 8 nickels

2. A triathlon athlete swims at an average rate 2.4 miles per hour, and bikes at an average rate of 16.1 miles per hour. At the end of one training session, she swam and biked more than 20 miles in total.

   The inequality $2.4s + 16.1b > 20$ and this graph represent the relationship between the hours of swimming, s, the hours of biking, h, and the total distance the athlete could have traveled in miles.



   Mai said, "I'm not sure the graph is right. For example, the point $(10, 3)$ is in the shaded region, but it's not realistic for an athlete to swim for 10 hours and bike for 3 hours in a training session! I think triathlon athletes generally train for no more than 2 hours a day."

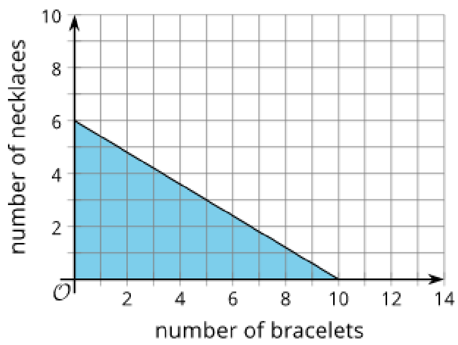
   a. Write an inequality to represent Mai's last statement.

   b. Graph the solution set to your inequality.

   c. Determine a possible combination of swimming and biking times that meet both the distance and the time constraints in this situation.

3. Elena is considering buying bracelets and necklaces as gifts for her friends. Bracelets cost \$3, and necklaces cost \$5. She can spend no more than \$30 on the gifts. Elena needs at least 7 gift items.

   This graph represents the inequality $3b + 5n \leq 30$, which describes the cost constraint in this situation. Let b represent the number of bracelets and n the number of necklaces.



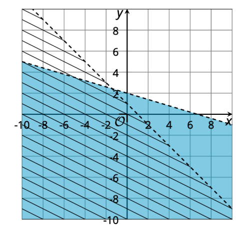
   a. Write an inequality that represents the number of gift items that Elena needs.

   b. On the same coordinate plane, graph the solution set to the inequality you wrote.

   c. Use the graphs to find at least two possible combinations of bracelets and necklaces Elena could buy.

   d. Explain how the graphs show that the combination of 2 bracelets and 5 necklaces meet one constraint in the situation but not the other constraint.

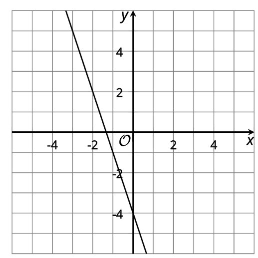
4. Two inequalities are graphed on the same coordinate plane.



   Which region represents the solution to the system of the two inequalities?

   (From Unit 3, Lesson 18)

5. Here is a graph of the equation $6x + 2y = -8$.



   a. Are the points $(1.5, -4)$ and $(0, -4)$ solutions to the equation? Explain or show how you know.

   b. Check if each of these points is a solution to the inequality $6x + 2y \leq -8$:

   $(-2, 2)$ $(4, -2)$ $(0, 0)$ $(-4, -4)$

   c. Shade the solutions to the inequality.

   d. Are the points on the line included in the solution region? Explain how you know.

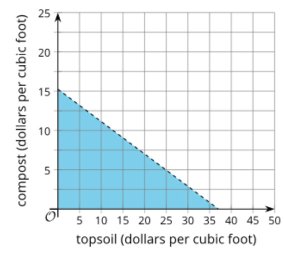
   (From Unit 3, Lesson 16)

6. A gardener is buying some topsoil and compost to fill his garden. His budget is \$70. Topsoil costs \$1.89 per cubic foot, and compost costs \$4.59 per cubic foot.

   Select **all** statements or representations that correctly describe the gardener's constraints in this situation. Let t represent the cubic feet of topsoil and c the cubic feet of compost.

   a. The combination of 7.5 cubic feet of topsoil and 12 cubic feet of compost is within the gardener's budget.

   b. If the line represents the equation $1.89t + 4.59c = 70$, this graph represents the solutions to the gardener's budget constraint.

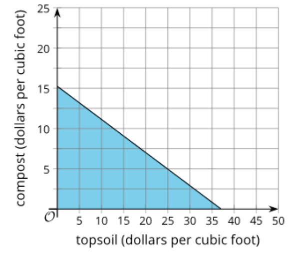


   c. $1.89t + 4.59c \geq 70$

   d. The combination of 5 cubic feet of topsoil and 20 cubic feet of compost is within the gardener's budget.

   e. $1.89t + 4.59c \leq 70$

   f. If the line represents the equation $1.89t + 4.59c = 70$, this graph represents the solutions to the gardener's budget constraint.



   (From Unit 3, Lesson 15)

7. Fill in the missing coordinates: S is the midpoint of segment UA, with points $U\,(6, y)$, $S\,(x, -4)$, and $A\,(-1, -1)$.

   (From Unit 3, Lessons 13 & 14)

8. Priya writes the equation $y = -\frac{1}{2}x - 7$. Write an equation that has:

   a. exactly one solution in common with Priya's equation

   b. no solutions in common with Priya's equation

   c. infinitely many solutions in common with Priya's equation, but looks different than hers

   (From Unit 3, Lesson 12)

9. Han is planning a trip to South Africa, and he wants to learn about rugby. It's one of the most popular sports in South Africa! In the first game Han watches, the South African National Team, the Springboks, scored 59 total points, with nine tries and seven conversions. Their opponent, the Wallabies from Australia, scored 49 total points, with nine tries and two conversions. If each try, *t*, and each conversion, *c*, is worth the same amount of points, how many points is one try and one conversion worth together?

   (From Unit 3, Lesson 10)

10. One side of a square has endpoints $A\,(1,4)$ and $B\,(4,0)$. What are the equations of the lines containing the two adjacent sides to side AB?

    (From Unit 3, Lesson 6)

11. Lin is taking a trip to Europe, and she needs to convert her American dollars to European currency. When she's in England, she'll need to use the currency, pounds, and when she's in France, she'll need to use the currency, euros. The current exchange rate is 0.71 pounds per dollar and 0.82 euros per dollar. Lin wants to bring $1,000 to spend.

    a. Write an equation describing the number of pounds, p, and euros, e, Lin can exchange for $1,000.

    b. If Lin exchanges the money in all pounds, how many will she get? Explain below how this could be represented on a graph.

    c. If Lin exchanges the money in all euros, how many will she get? Explain below how this could be represented on a graph.

    (From Unit 3, Lesson 2)

Lesson 20: Modeling with Systems of Inequalities in Two Variables

PREPARATION

Lesson Goals	Learning Targets
• Define the constraints in a situation and create a mathematical model to represent them. • Interpret a mathematical model, presented as inequalities and graphs, that represents a situation.	• I can interpret inequalities and graphs in a mathematical model. • I know how to choose variables, specify the constraints, and write inequalities to create a mathematical model for a given situation.

Lesson Narrative

In this culminating lesson, students integrate the ideas from the unit and engage in multiple aspects of mathematical modeling (MP4).

In the first activity, they interpret and analyze given models that represent the constraints and conditions in a situation. In the second, optional activity, they create their own models after specifying quantities of interest, identifying relevant information, and setting the constraints. There is particular focus on including constraints that are not given in the problem statement but can be inferred from context: namely, if the values of both variables must be positive, then the constraints $x > 0$ and $y > 0$ also belong in the system.

Share some ways you see this lesson connecting to previous lessons in this unit. What connections will you want to make explicit?

Focus and Coherence

Addressing
NC.M1.A-CED.3: Create systems of linear equations and inequalities to model situations in context.
NC.M1.A-REI.12: Represent the solutions of a linear inequality or a system of linear inequalities graphically as a region of the plane.

Agenda, Materials, and Preparation

- **Warm-up** (*5 minutes*)
- **Activity 1** (*20 minutes*)
- **Activity 2** (*10 minutes*)
  - Technology is required for this activity: Acquire devices that can access Desmos (recommended) or other graphing technology. It is ideal if each student has their own device.
  - Blank visual displays for each student group (possible visual display options: poster board, chart paper, Google Slides, Jamboard)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U3.L20 Cool-down (print 1 copy per student)

LESSON

Warm-up: A Solution to Which Inequalities? (*5 minutes*)

Addressing: NC.M1.A-REI.12

This warm-up gives students a quick exposure to the inequalities $x > 0$, $x \geq 0$, $y > 0$, and $y \geq 0$, so that they are prepared to deal with them later in this lesson. It also reinforces the idea of thinking carefully about whether the points on the boundary lines of a solution region are included in the solution set.

Step 1

- Provide students with quiet think time to reflect on the warm-up.

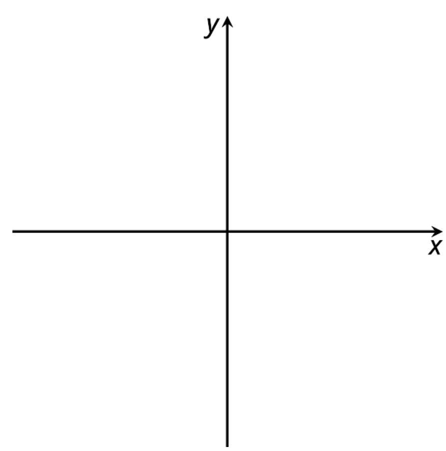
Student Task Statement

Is the ordered pair $(5.43, 0)$ a solution to all, some, or none of these inequalities? Be prepared to explain your reasoning.

$x > 0$ $\quad$ $y > 0$ $\quad$ $x \geq 0$ $\quad$ $y \geq 0$

Step 2

- Invite students to share their responses.
- Display a blank four-quadrant coordinate plane for all to see.
- Ask students:
  - "If we were to graph the solutions to $x > 0$, what would the region look like?" (We would shade the right side of the y-axis.)
  - "Is the y-axis included in the solution region?" (No)
  - "What about the graph of the solutions to $y > 0$?" (We would shade the upper side of the x-axis.)
  - "Is the x-axis included in the solution region?" (No)
  - "What about the graph of the solutions to the system $x > 0$ and $y > 0$?" (The solution region would be the upper-right section of the graph, where the other two regions overlap.)



Remind students that this upper-right region of the coordinate plane is called *the first quadrant*.

DO THE MATH	PLANNING NOTES

Activity 1: Custom Trail Mix (*20 minutes*)

Instructional Routines: Aspects of Mathematical Modeling; Notice and Wonder; Collect and Display (MLR2)
Addressing: NC.M1.A-REI.12

In this activity, students engage in *Aspects of Mathematical Modeling* by using their insights from the unit to analyze and interpret a set of mathematical models and a set of data in context. Each situation involves more than two constraints, and can therefore be represented with a system with more than two inequalities.

Interpreting and connecting the inequalities, the graphs, and the data set (which involves decimals) prompts students to make sense of problems and persevere in solving them (MP1), and to reason quantitatively and abstractly (MP2).

Step 1

- Ask students to arrange themselves into pairs or use visibly random grouping.
- Give students a minute to skim the task to familiarize themselves with the given information.
- Using the *Notice and Wonder* routine, ask students for one thing they noticed and one thing they wondered as they looked over the task.

Step 2

- Ask students to use the table to write an expression to represent the total amount of fiber if they had a grams of almonds and b grams of raisins. Students should see that the expression is $0.07a + 0.05b$.
- Next, ask for an expression representing the total amount of sugar for the same amounts of almonds and raisins ($0.21a + 0.60b$).
- Ask students to analyze and answer the questions about one student's trail mix (either Tyler's or Jada's). If time permits, the groups could analyze the other trail mix.
- Provide students 5 minutes of individual quiet work time and then five minutes to share their thinking with their partner.

RESPONSIVE STRATEGY

Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, highlight values from the table, equations, and graphs in corresponding colors as they discover the connections. Encourage students to label regions of the graph and label variables in addition to color coding to reinforce connections.

Supports accessibility for: Visual-spatial processing

Use the *Collect and Display* routine as you circulate and listen to partner conversations. Capture any language students use to describe the inequalities and the graphs, including both everyday and mathematical language students have been working with throughout this unit.

Student Task Statement

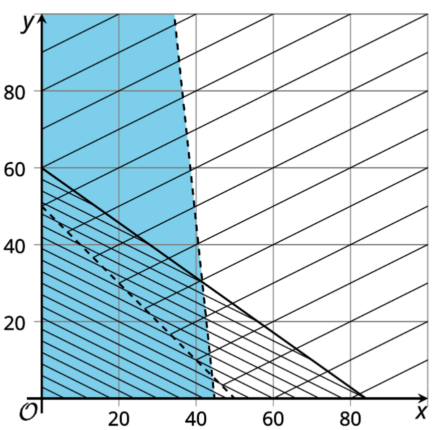
Here is the nutrition information for some trail mix ingredients:

	calories per gram (kcal)	protein per gram (g)	sugar per gram (g)	fat per gram (g)	fiber per gram (g)
peanuts	5.36	0.21	0.04	0.46	0.07
almonds	5.71	0.18	0.21	0.46	0.07
raisins	3.00	0.03	0.60	0.00	0.05
chocolate pieces	4.76	0.05	0.67	0.19	0.02
shredded coconut	6.67	0.07	0.07	0.67	0.13
sunflower seeds	5.50	0.20	0.03	0.47	0.10
dried cherries	3.25	0.03	0.68	0.00	0.03
walnuts	6.43	0.14	0.04	0.61	0.07

Tyler and Jada each designed their own custom trail mix using two of these ingredients. They wrote inequalities and created graphs to represent their constraints.

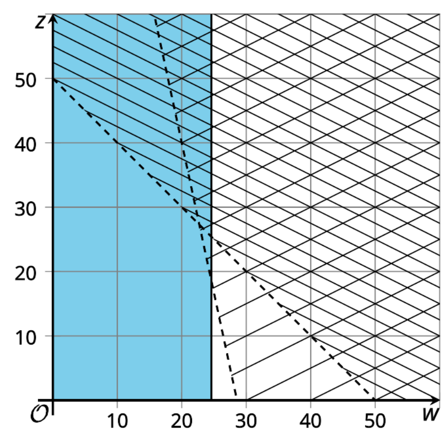
Tyler

- $x + y > 50$
- $4.76x + 6.67y \le 400$
- $0.67x + 0.07y < 30$
- $x > 0$
- $y > 0$



Jada

- $w + z > 50$
- $0.14w + 0.03z > 4$
- $0.61w + 0z \le 15$
- $w > 0$
- $z > 0$



Use the inequalities and graphs to answer these questions about each student's trail mix. Be prepared to explain your reasoning.

1. Which two ingredients did they choose?
2. What do their variables represent?
3. What does each constraint mean?
4. Which graph represents which constraint?
5. Name one possible combination of ingredients for their trail mix.

Step 3

- Display a copy of both Tyler's and Jada's equations and graphs, and annotate the display during the class discussion. Students should have their workbooks open to the nutritional information table to reference. As you hear correct vocabulary or phrases when discussing the questions below, link these to the display, and invite students to borrow any relevant language that was collected while students were working on the task. Use this opportunity to refine any language that is still imprecise by asking students to communicate with words and phrases that everyone in the room understands.

- Facilitate a discussion focused on the connections between the graphs and the inequalities, and on the inequalities $x > 0$ and $y > 0$. Ask questions such as:
  - "How did you know which ingredients each person used?" (by matching the coefficients in two of the inequalities to the nutritional values in the table)
  - "The table shows the same values for some nutrients. How can you tell which one Tyler or Jada chose?" (The coefficients of x and y in one inequality and those in the other inequality must be for the same two ingredients.)
  - "Why do you think Jada and Tyler both included the inequalities $x > 0$ and $y > 0$?" (There cannot be only one ingredient, so both x and y must be greater than 0.)
  - "How do those inequalities affect the graph of the solution region?" (They limit the solution region to the first quadrant.)
  - "Jada and Tyler each wrote five inequalities. Could all five form a single system?" (Yes) What does it mean to have a system with five inequalities?" (There are five constraints that must be met. The solutions to the system satisfy all five constraints simultaneously.)

DO THE MATH	PLANNING NOTES

Activity 2: Design Your Own Trail Mix (*10 minutes*)

Instructional Routines: Aspects of Mathematical Modeling; Compare and Connect (MLR7)
Addressing: NC.M1.A-CED.3, NC.M1.A-REI.12

This activity is designed to give students opportunities to use their understanding from this unit to perform mathematical modeling.

The trail mix context is familiar from the previous activity, but students are challenged to choose quantities, determine how to represent them, interpret and reason about them, and use the model they create to make choices. It also enables students to reflect on their model and revise it as needed (MP4).

Students are likely to want to use graphing technology, as the nutritional information involves decimals and the inequalities written would be inconvenient to graph by hand. This is an opportunity for students to choose tools strategically (MP5).

Step 1

- Ask students to arrange themselves into small groups or use visibly random grouping.
- Provide access to Desmos or other graphing technology.
- Explain the expectations for
  - researching nutritional values,
  - number of constraints necessary,
  - collaboration with group members, and
  - for presentation of student work. (If each group is presenting one response, provide each group with tools for creating a visual display. If each student is presenting a response, give each student tools for creating a visual display.)

RESPONSIVE STRATEGIES

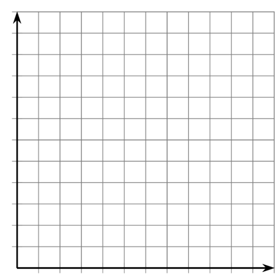
Provide prompts, reminders, guides, rubrics, or checklists that focus on increasing the length of on-task orientation in the face of distractions. For each question, allot a specific amount of work time and display expectations. Display a countdown timer, along with a bulleted list of what teams or individuals are to produce to complete a given step. For instance, during question 1, display a 5 minute countdown timer and the list: “You’re finished if you. . . (1) Have your two ingredients (2) Have the nutrition information ready.”

Supports accessibility for: Attention; Social-emotional skills

Student Task Statement

It's time to design your own trail mix!

1. Choose two ingredients that you like to eat. (You can choose from the ingredients in the previous activity, or you can look up nutrition information for other ingredients.)
2. Think about the constraints for your trail mix. What do you want to be true about its calories, protein, sugar, fat, or fiber?
3. Write inequalities to represent your constraints. Then, graph the inequalities.
4. Is it possible to make a trail mix that meets all your constraints using your ingredients? If not, make changes to your constraints or your ingredients and record them here.
5. Write a possible combination of ingredients for your trail mix.



Step 2 (*Optional*)

- If time allows, have students create a display of their work.
- Select groups to share their visual displays. Give students 2–3 minutes of quiet think time to interpret the displays before inviting the authors to present their work.
- Use the *Compare and Connect* routine to invite students to ask questions about the mathematical thinking or design approach that went into creating each display. Model this process by asking questions such as:
  - What constraints were used? Did some groups use the same?
  - How do the graphs of the various mixes compare?
  - Why is it important that $x > 0$ and $y > 0$ are constraints?
  - Did your group revise or change their model in order to come up with a solution they could use?

- How did you use the graph to choose a recipe for your mix?
- How are the group trail mixes similar and different?
- How was each group's mathematical approach to making trail mix similar and different?

DO THE MATH

PLANNING NOTES

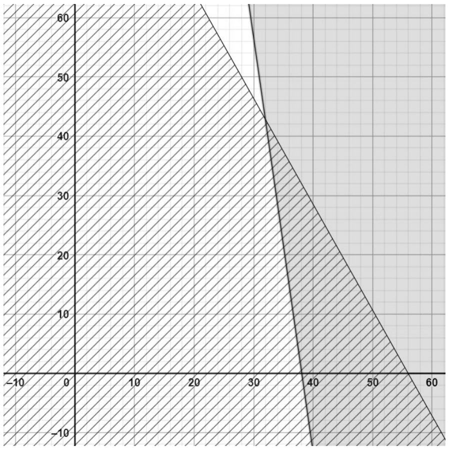
Lesson Debrief (*5 minutes*)

In this lesson, students have analyzed systems in context. These systems included more than two constraints with a focus on restricting solution sets to the first quadrant. Facilitate a discussion using the following questions:

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

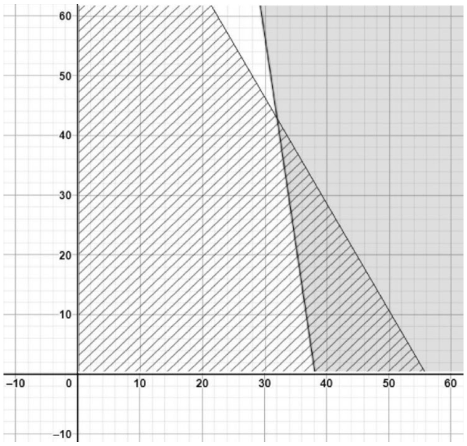
Consider a trail mix of raisins and peanuts. Display the two systems.

System A



$$\begin{cases} 5.36x + 3y \leq 300 \\ 0.21x + 0.03y \geq 15 \end{cases}$$

System B



$$\begin{cases} 5.36x + 3y \leq 300 \\ 0.21x + 0.03y \geq 15 \end{cases}$$

PLANNING NOTES

- "How are the graphs similar? How are they different?" (Both systems have the same two constraints. System B is restricted to the first quadrant.)
- "Identify a point that is a solution to system A but not system B. Explain how you determined the point." (Sample Response: (50,-5) is located within the overlapping regions for system A, meaning it is a solution to system A. In system B, the values below the x-axis are no longer included so the point is not a solution for system B.)
- What are the missing constraints for System B? ($x > 0$ and $y > 0$)
- Which system is a better model for the trail mix containing raisins and peanuts? (System B is a better model because the grams of raisins and grams of peanuts can be any value greater than 0.)

Student Lesson Summary and Glossary

Each day a small bakery bakes two types of bread, A and B.

- One batch of bread A uses 5 pounds of oats and 3 pounds of flour.
- One batch of bread B uses 2 pounds of oats and 3 pounds of flour.
- The company has 180 pounds of oats and 135 pounds of flour available each day.

What are the constraints in this situation?

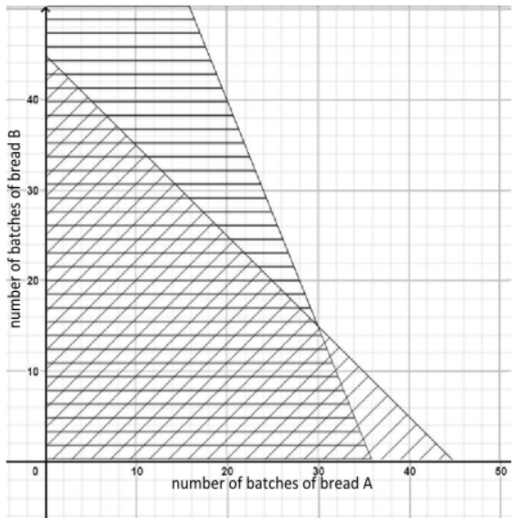
- There is a limit on the amount of oats (180 pounds).
- There is a limit on the amount of flour (135 pounds).
- The number of batches of bread A and bread B must be positive.

Let a represent the number of batches of bread A and let b represent the number of batches of bread B. The constraints can be represented with a system of inequalities.

- $5a + 2b \leq 180$
- $3a + 3b \leq 135$
- $a > 0$
- $b > 0$

Here are the graphs of the inequalities in the system. Notice how the solution set is restricted to the first quadrant.

The constraints of $a > 0$ and $b > 0$ are necessary in this situation for two reasons. First, the number of batches of bread must be a positive number thus greater than 0. Secondly, the bakery will bake both types of bread each day so neither could be equal to 0.



Cool-down: Making Bracelets (*5 minutes*)

Addressing: NC.M1.A-CED.3; NC.M1.A-REI.12

Cool-down

Andre can make bracelets in two sizes, large and small. The table shows how much material each bracelet requires. Andre has 5 yards of leather and 6 yards of string. (1 yard is equivalent to 36 inches.)

Size	Leather needed	String needed
Large bracelet	4 inches	18 inches
Small bracelet	12 inches	3 inches

1. If x represents the number of small bracelets and y represents the number of large bracelets, which system of inequalities models the constraints on the number of bracelets Andre can make? Label the constraints of the correct system of inequalities.

a.
$$\begin{cases} 12x + 4y \leq 180 \\ 3x + 18y \leq 216 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

b.
$$\begin{cases} 12x + 3y \leq 180 \\ 4x + 18y \leq 216 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

c.
$$\begin{cases} 12x + 4y \leq 5 \\ 3x + 18y \leq 6 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

d.
$$\begin{cases} 12x + 3y \leq 5 \\ 4x + 18y \leq 6 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

2. Explain what $x \geq 0$ and $y \geq 0$ mean about Andre's situation.

Student Reflection: How does technology help you understand the mathematical concepts you learned today? Do you think your understanding would be different without the use of technological tools?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Which students came up with an unexpected strategy in today's lesson? What are some ways you can be more open to the ideas of each and every student?

Practice Problems

1. Four families are visiting Carowinds. For each family, let x be the amount of time spent riding rides and let y be the time spent in Carolina Harbor Waterpark. List one or more ordered pairs that would fit the constraints given by each family.

   a. Clare's family wants to spend at least 4 hours riding the rides, and they want to spend more time in the Carolina Harbor Waterpark than riding rides.

   b. Jada's family wants to be at Carowinds from 4 p.m. to 8 p.m., and they want to spend most of their time riding rides.

   c. Priya's family wants to spend 2 hours at Carolina Harbor Waterpark and 2 hours riding rides.

   d. Diego's family wants to spend no more than 6 hours at Carowinds, and they want to spend at least twice as long riding rides as they spend at Carolina Harbor Waterpark.

2. The organizers of a conference need to prepare at least 200 notepads for the event and have a budget of $160 for the notepads. A store sells notepads in packages of 24 and packages of 6.

   This system of inequalities represent these constraints: $\begin{cases} 24x + 6y \geq 200 \\ 16x + 5.40y \leq 160 \end{cases}$

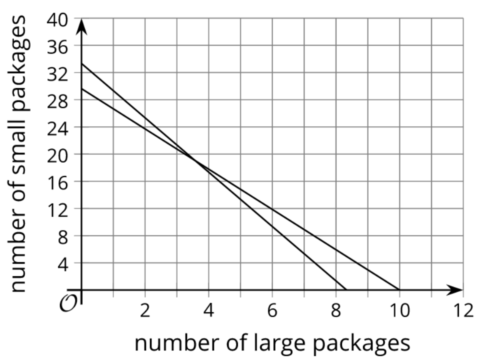
   a. Explain what the second inequality in the system tells us about the situation.

   b. Here are incomplete graphs of the inequalities in the system, showing only the boundary lines of the solution regions.

      Which graph represents the boundary line of the second inequality?

   c. Complete the graphs to show the solution set to the system of inequalities.

   d. Find a possible combination of large and small packages of notepads the organizer could order.



   (From Unit 3, Lesson 19)

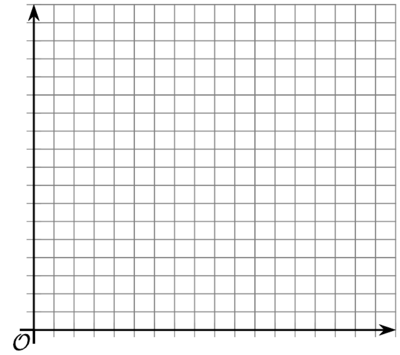
3. A certain stylist charges $15 for a haircut and $30 for hair coloring. A haircut takes on average 30 minutes, while coloring takes 2 hours. The stylist works up to 8 hours in a day, and she needs to make a minimum of $150 a day to pay for her expenses.

    a. Create a system of inequalities that describes the constraints in this situation. Be sure to specify what each variable represents.

    b. Graph the inequalities and show the solution set.

    c. Identify a point that represents a combination of haircuts and hair-coloring jobs that meets the stylist's requirements.

    d. Identify a point that is a solution to the system of inequalities but is not possible or not likely in the situation. Explain why this solution is impossible or unlikely.

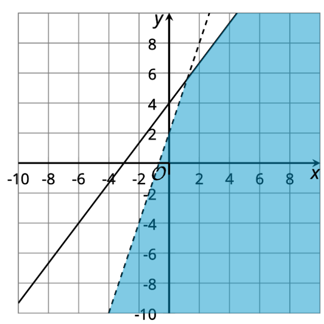


(From Unit 3, Lesson 19)

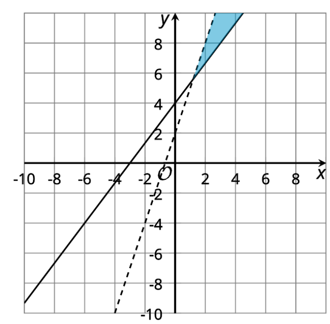
4. Choose the graph that shows the solution to this system:

$$\begin{cases} y > 3x + 2 \\ -4x + 3y \leq 12 \end{cases}$$

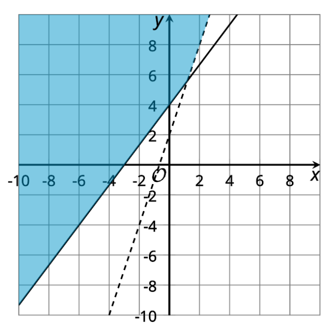
a.



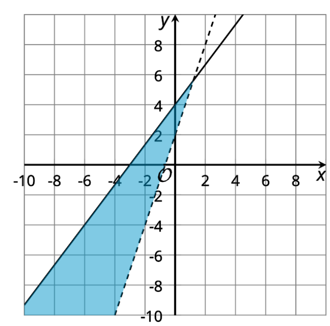
b.



c.

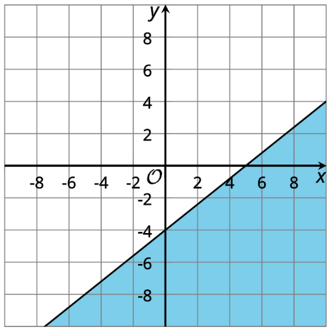
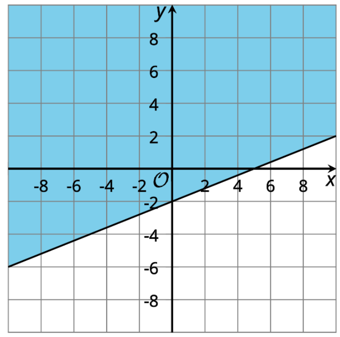
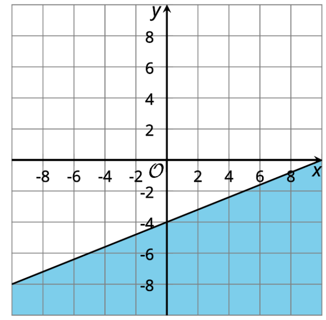
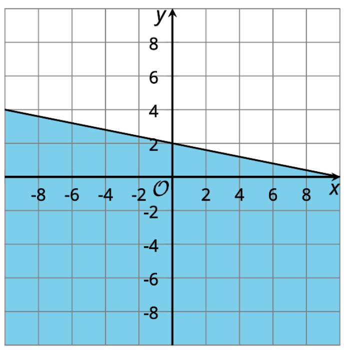
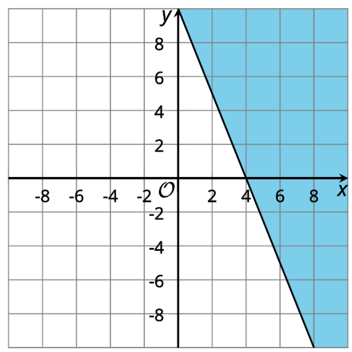


d.



(From Unit 3, Lesson 18)

5. Match each inequality to the graph of its solution.

Inequality	**Graph**
1. $2x - 5y \geq 20$ 2. $5x + 2y \geq 20$ 3. $4x - 10y \leq 20$ 4. $4x - 5y \geq 20$ 5. $2x + 10y \leq 20$	a.  b.  c.  d.  e. 

(From Unit 3, Lesson 17)

6.

a. In a system of linear equations, the equation for one line is $y = -4.7x + 6.2$. Write a possible equation for the other line if the system has no solutions.

b. In a system of linear equations, the equation for one line is $y + \frac{7}{2} = 2(x - \frac{5}{2})$. Write a possible equation for the other line if the system has one solution.

c. In a system of linear equations, the equation for one line is $5x + 6y = 7$. Write a possible equation for the other line if the system has infinite solutions.

(From Unit 3, Lesson 12)

7. A quadrilateral has vertices $A = (0, 0), B = (4, 6), C = (0, 12)$, and $D = (-4, 6)$. Mai thinks the quadrilateral is a rhombus and Elena thinks the quadrilateral is a square. Do you agree with either of them? Show or explain your reasoning.

(From Unit 3 Lesson 8)

8. Diego and Clare are both saving up for prom - between the ticket, clothes, and dinner, it can get expensive! On March 1, Diego has $30 and can make $14.75 an hour at his part time job, and Clare has $25 and can make $14.75 an hour at her part time job.

    a. Write an equation to represent how much money, m, Diego and Clare will each have after working h hours at their jobs.

    b. Clare thinks they'll have the same amount of money after each of them works 35 hours. Do you agree? Why or why not?

    c. How can a graph of the equations prove whether Clare is correct or not?

    (From Unit 3, Lesson 5)

9. Lin's father sends her to the store with $40 to buy balloons and cupcakes for her sister's birthday party. Balloons cost $1.50 each, and cupcakes are sold in packs of 6 for $3.50.

    a. Write an equation to represent how many balloons, b, and packs of cupcakes, c, Lin can buy with $40.

    b. Lin returns home with 30 cupcakes and 15 balloons. Should Lin have change for her father? How do you know?

    (From Unit 3, Lesson 1)

Lesson 21: Post-Test Activities

PREPARATION

Lesson Goals	Learning Targets
• Communicate high expectations for all students. • Build a welcoming classroom community that recognizes and values the unique perspectives and experiences each student brings.	• I understand the reasoning for and will strive to meet the expectations communicated by my teacher. • I know my classmates and can recognize the value I will add to this classroom community.

Lesson Narrative

This lesson, which should occur after the Unit 3 End-of-Unit Assessment, allows for students to reflect on the unit, share feedback, conference with the teacher, and engage in a fun community-building activity.

Gathering student feedback is a powerful and strategic way to learn about students and improve instructional practices. It also creates student and family buy-in and centers students as decision makers and problem solvers in their own learning.

Conducting one-on-one conferences with students, using the surveys as a data point, is encouraged. These conferences can be done as students are engaging in Activity 2. Potential conference topics include:

- student responses to the daily student reflections
- student response to the end-of-unit student survey (as students finish them)
- executive functioning skills
- student learning contracts
- goal setting and self-evaluation

What do you hope to learn about your students during this lesson?

Agenda, Materials, and Preparation

- **Activity 1** (*20 minutes*)
  - End-of-Unit 3 Student Survey (print 1 copy per student)
- **Activity 2** (*25 minutes*)
  - Culture and Mathematics articles (print multiple copies, separate and staple individual articles for students to choose from)
  - Tools for each student for creating a visual display: for example, construction paper, chart paper, whiteboard space, shared online drawing tool, or access to a document camera
  - Markers or post-it notes for commenting on displays

LESSON

Activity 1: End-of-Unit 3 Student Survey (*20 minutes*)

The End-of-Unit 3 Student Survey is a critical opportunity for teachers to gather low-stakes, non-evaluative feedback to support equity and instructional pedagogy. The survey is also highly beneficial for students as it is designed to encourage self-awareness, self-management, social awareness, relationship skills, and responsible decision making. Provide students a chance to quietly and independently complete this survey after they complete their testing.

One-on-One Conferences

Conducting one-on-one conferences with students, using the surveys as a data point, is encouraged. These conferences can be done as students complete their surveys and are engaging in Activity 2. Potential conference topics include:

- student responses to the daily student reflections
- student response to the end-of-unit student survey (as students finish them)
- executive functioning skills
- student learning contracts
- goal setting and self-evaluation

Activity 2: Culture and Mathematics (*25 minutes*)

This activity is intended to help students see the ways an activity they engage in with their culture, family, and/or community connects to mathematics.

- Provide students access to the culture and mathematics articles, tools for creating a display, and tools for sharing feedback to classmates.

Student Task Statement

Take a moment to think about a few of the ways in which your friends, family, and/or community interact with each other. Create a list or share with a partner.

Many cultures are strengthened through interactions revolving around food, music, art, gaming/play, dancing, sports. and public policy.

During this activity you will complete three sections:

1. Discover
    a. Choose one of the following subjects to explore: food, music, art, gaming/play, dancing, sports, or public policy.
    b. Brainstorm the ways in which your family, friends, and/or community engage in the subject you chose.
    c. Read the related article to understand the connections between your cultural interactions and mathematics. Take notes below.
2. Share
    a. Pair up with one or two classmates who chose the same subject area.
    b. Discuss:
        i. One way you engage with this activity that you'd like to share.
        ii. One thing you learned from the article.
        iii. One thing you are curious to learn more about around this activity and its connection to math.
    c. Create one chart that represents your group's discussion and display it in the classroom.

3. Learn and Affirm
    a. Walk around the classroom to discover the ways your classmates engage in their friends, families, and/or communities.
    b. Use markers or post-it notes to leave positive notes to your classmates about what they shared.
    c. Reflect: share with a partner or write in your workbook about one thing you found interesting from what your classmates shared.

TEACHER REFLECTION

As you finish up this unit, reflect on the norms and activities that have supported each student in learning math. List ways you have seen each student grow as a young mathematician throughout this work.

List ways you have seen yourself grow as a teacher.

What will you continue to do, and what will you improve upon, in Unit 4?